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EXECUTIVE SUMMARY

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This work examines a new model developed to estimate proportions of each cohort and sex from a length frequency distribution of hoki (*Macruronus novaezelandiae*). The model is a generalised version of MULTIFAN in that it estimates cohort specific growth curves and proportions at age, but is able to incorporate two stocks and estimate cohort and year effects. The proportions of each cohort and sex in the commercial catch from the Chatham Rise and Sub-Antarctic for the fishing year 1991–92 to 1999–2000 are presented.

To estimate the growth of a cohort and alleviate the problem of growth occurring over the period of sampling, the observed commercial tows from the Chatham Rise and Sub-Antarctic were separated into short time periods in which minimal growth occurs. Each time period was further stratified by depth, latitude, longitude, and/or time of tow (day or night), depending on the number of samples available. Tree-based regression techniques were used to find the most important stratification variables. The Chatham Rise showed tendencies to stratify at depths of 350–400 m and 500–600 m. Latitude splits occurred near the east to west 500 m depth contours, but never split the Chatham Rise into north and south halves. Longitude was an occasional stratification variable, occurring near 176° E and 179° E. The Sub-Antarctic generally attempted to split the observed lengths into naturally defined areas around Pukaki Rise, the Auckland Islands, Campbell Rise, and the Snares south of Stewart Island. Mean lengths at age determined from trawl surveys in the two areas were also included in the model.

Models for the Chatham Rise and Sub-Antarctic were separately fitted to observer length frequency and mean length at age data. The data for the two areas were then combined into one model and proportions at age and sex were estimated for the commercial catch in each area. The 1997 cohorts from both areas showed faster growth rates and significant year effects occurred in 1991, 1998, and 1999. The 1987, 1988, 1991, 1992, 1994, and 1997 cohorts were consistently represented in the catch with a high proportion suggesting they are strong cohorts. Proportions of each cohort and sex in the commercial catch for the fishing years 1991–92 to 1999–2000 are reported for each area.

This new method for predicting the proportion at age and sex in the commercial catch is a significant improvement over the previously used MIX analysis, mostly in that the incorporation of growth curves reduces some of the subjectivity present when deciding on mean lengths at age. However, assumptions are still made in the model and using least squares as the objective function may not be the most appropriate way to model the length frequencies. Some ideas that may improve the model are given.

1. INTRODUCTION

Hoki (*Macruronus novaezelandiae*) are managed in New Zealand under the theory that there are two stocks, each of which migrates between a spawning and home ground. The two spawning grounds are located off the west coast of the South Island (WCSI) and in Cook Strait (CKST). Hoki which belong to the WCSI stock (western stock) are believed to make their way to the Chatham Rise as juveniles then migrate to their home ground in the Sub-Antarctic. The hoki from Cook Strait (eastern stock) migrate to their home ground, the Chatham Rise, as juveniles (Annala et al. 2001).

A single TACC is set for hoki which applies to both stocks (Ballara et al. 2000). Historically, the main fisheries have taken place during the spawning season from late June to mid September around the west coast of the South Island and in Cook Strait, but in the last decade year round fisheries have developed on the Chatham Rise and in the Sub-Antarctic region (Figure 1). The Scientific Observer Programme (SOP) places observers on commercial fishing vessels to sample lengths and other information from the hoki commercial catch. These data are then used to compile length frequencies and calculate proportions at age and sex for input into the stock assessment model.

The proportions of each cohort in the commercial catch from the non-spawning areas have been calculated using MIX software (Ballara & Livingston 2001). However, the year round fisheries on the Chatham Rise and Sub-Antarctic result in the collection of fish from the same cohort that have grown during the year, thus blurring the length modes when the data are aggregated, even though they may be well defined when sampled over a short period of time. Another problem of the MIX analysis is that the mean lengths at age are subjectively determined and may not be consistent from year to year. To overcome these problems, a model combining the benefits of the MULTIFAN model (Fournier et al. 1990) and MIX was developed to estimate the proportion of each cohort from multiple length frequency samples spread over time. Partitioning the observed length frequencies into short time periods in which minimal growth occurs allows cohort specific growth curves to be estimated and used in the calculation of proportions at age.

The primary goal of this study was to determine if this model can improve upon the estimates of the proportion of each cohort and sex in the commercial catch of hoki from the two major non-spawning areas, the Chatham Rise and the Sub-Antarctic. Proportions of each cohort and sex from the two non-spawning fisheries are reported for the fishing years 1991–92 to 1999–2000 and compared to the previous results from the MIX analysis. A by-product of this analysis is the identification of specific years and/or cohorts that have different growth patterns.

2. DATA INPUTS

The data inputs into this model can consist of length frequencies obtained from trawl surveys, the Scientific Observer Programme and the stock monitoring programme, as well as mean lengths at age obtained from trawl surveys. The Chatham Rise and Sub-Antarctic fisheries occur during much of the year, thus length frequencies from the commercial catch in these areas were produced for various time periods within the year. The time periods were chosen based on the number of tows sampled and assumptions of growth.

2.1 Length frequencies

Sub-Antarctic and Chatham Rise observer data from the fishing years 1992–2000 were used in this analysis. These years were chosen because the observer coverage before 1992 was minimal and length frequency samples were not as temporally spread out.

It is important to post-stratify length frequency samples to increase precision and avoid biases due to variation in length or non-representative sampling (Bradford 2000, Bull & Gilbert 2001). The stratification chosen should be reasonable and reflect genuine differences between strata. Otherwise, low within-stratum variation can lead to an underestimate of the total variance (Bull & Gilbert 2001). If possible, exogenous information should be used to help determine justifiable strata.

Bradford (2000) reported that the Chatham Rise observer coverage tends to be clumped in time, not cover the eastern portion of the rise, and not cover the shallow and deep depths. Appropriate post-stratification is suggested based on these variables.

Length frequencies for this analysis were separately compiled from observer data collected in the Chatham Rise and Sub-Antarctic areas. These frequencies represent short time intervals where, it is hoped, not much growth occurs. Therefore, 4–8 week periods in the summer, and 4–12 week periods in the winter were determined based on the number of samples collected within a fishing year. Time strata were chosen so that the samples would be representative of the mid-point within the period, thus long periods of time with zero samples were excluded from the stratification. The actual time strata used were determined *ad hoc* such that a significant number of samples occurred within each stratum, contained by the number of weeks stated above. This eliminates the need to stratify further based on time.

Within each time period, the observed lengths were stratified based on the variables latitude, longitude, depth of the net, time of day (night or day), and fishing method (midwater or bottom). All these variables were determined from the start of the tow. The time of day was classified as night or day based the 2001 sunset/sunrise times of the nearest location published by Land Information New Zealand (2001). A tow was classified as "day" if the starting time of the tow was after sunrise and before sunset, and classified as "night" otherwise. The fishing method was midwater if the difference between the depth of the net and the bottom depth was greater than 10 m. The choice of stratification within each time period deals with the non-representative observer coverage reported by Bradford (2000). However, vessel length and vessel nation were not included as stratification variables, as we assumed that the time periods are short enough such that the number of different vessels fishing within each time period is small, thus removing large variations in the vessel characteristics. Due to the difficulty of accurately determining vessel characteristics, this assumption was not tested.

Francis (2002) used tree-based regression to help determine strata on the Chatham Rise that would explain the most amount of variation present in observer length frequency samples from the fishing years 1997–98, 1998–99, and 1999–2000. This method begins with all the samples in one stratum and determines where to divide the sample, based on one of the variables, to minimise the proposed stratified variation. This variation is calculated as

$$\sum_j (l_{ij} - \bar{l}_j)^2$$

where l_{ij} is the mean length from the i^{th} sample in stratum j, and $\bar{l}_j$ is the overall mean length in stratum j. New strata are then split from existing strata until a stopping point is reached. This stopping point can be defined by a minimum amount of variation explained, a minimum number of samples in a stratum, and a maximum "depth" (number of levels) to split. Francis (2002) used latitude, longitude, depth of the net, and the day of the fishing year as splitting variables.

Tree-based regression was first used on all the observer data within the fishing year (not split into time periods) and area (Chatham Rise or Sub-Antarctic) to determine likely strata divisions. The final stratification was determined by separately applying the tree-based regression to non-sex-specific mean lengths in each period, then reviewing the results to determine if the proposed strata were consistent with previous research, the analysis of all the data, or seemed likely because of natural features which could separate sub-populations of hoki, such as deep channels. Another method of stratification is to first determine overall strata, then apply these stratifications within each time period. However, each time period may show variation which is masked when analysing all of the data, therefore were separately stratified in this analysis.

The splitting algorithm was stopped when the sample size in each stratum was less than 10 or the amount of additional variation explained, when compared to the variation of the unstratified data, was less than 1%. As in Francis (2002), the amount of additional variance explained was calculated as

$$100\left(\frac{S_k - S_{k+1}}{S_0}\right)$$

where S_k is the sum of squares after the k^{th} split. Strata could contain less than 10 samples if a division was strongly supported by the above criteria and each area contained more than 5 samples. A sample is defined as a tow from which 100 hoki were typically measured.

Deciding on the final stratification within a time period is subjective and variability can be caused by outliers in the mean lengths, especially when using small sample sizes. Each splitting variable chosen by the tree-based regression was carefully scrutinised to make sure that a single observation did not influence the split. Therefore, sometimes a splitting variable was chosen which did not minimise the variation between the newly chosen strata, but explained a fair amount of the variation and made more sense than perhaps the haphazard variable chosen by the splitting algorithm. If no further division was suitable, the stratification was stopped even if the sample size was greater than 10 and the split explained more than 1% of the variation. Using tree-based regression on small sample sizes should be done with caution as normal random fluctuations can cause misleading splits and a single outlier may lead to a stratification which attempts to isolate that observation.

2.1.1 Chatham Rise stratification

A large portion of the annual hoki catch is taken from the Chatham Rise, sometimes surpassing the catch from Cook Strait. Tows since the 1991–92 fishing year that have had hoki lengths observed stretch from the 500 m depth contour nearest to the South Island out to the 1000 m depth contour at the eastern end of the rise (Figure 2). The stratification of the Chatham Rise region for the estimation of length frequencies has been the subject of recent analyses (Bradford 2000, Francis 2002). These analyses all suggest that the observed lengths should be post-stratified to avoid biases due to differences in mean length and unrepresentative observer sampling. Bradford (2000) found that depth and longitude showed some unrepresentativeness in sampling. Francis (2002) defined strata using depth and longitude.

The Chatham Rise observer sampling programme was sporadic from 1992 through 1997. However, the number of tows sampled increased greatly in the 1997–98 and 1998–99 fishing years with an even coverage over time. Recently, sampling seems to cover time relatively well. Time periods for this model were chosen such that they contained a cluster of samples representing a short period of time in which minimal growth occurs. Sometimes, these periods would be padded on either side by weeks with no sampling. For example, between 28 January and 24 February 1992, 37 length frequency samples were collected, but the three weeks before and after this period contained no sampling. Therefore, the time period was defined as 1 January to 17 March 1992. The median date remains the same and it makes no difference to the model. Some periods will appear to be rather long, but most likely contain no sampling at the start and end. Table 1 shows the median date and the number of weeks in each time period chosen for the Chatham Rise area.

The tree-based regression on each of the separate fishing years suggested that depth is a very important stratification variable. It typically made depth splits around 350–400 m and/or 500–600 m with larger fish occurring deeper. Latitude occasionally came in as a strong variable to stratify on, but it never split the rise into symmetric north and south halves. Values for latitude were generally near 43° S or 44° S, which is about where the 500 m depth contour runs from west to east. It may be that latitude can act as a proxy for depth, but it is also possible that the catches on the northwest or southwest part of the Chatham Rise are being separated out. Longitude was also an important variable with splits usually separating the rise at 176° E and 179° E. Larger fish usually were found east of the split. In 1995, some catches of very large fish occurred east of 175° W. Overall, depth and longitude were promising splitting variables, with latitude possibly used as a splitting variable on the northern or southern part of the rise.

Tree-based regression was used to determine initial strata within each time period. The final strata were then decided on using those results and reasonable judgement of suitable strata. Two to four strata was common for most time periods, with a few periods having more strata defined if there was an ample number of tows. Some variation was explained by the time of day, and was used in 3 of the 53 time periods. Fishing method was not used as a splitting variable because there usually were not enough samples using both methods, but depth may be a good proxy for the differences between midwater and bottom gear. A latitude split near 44° S was common in the 1999–2000 fishing year, possibly because more tows were observed in the southwestern portion of the rise during this year than in other years.

2.1.2 Sub-Antarctic stratification

Observations from the Puysegur area off the southwestern end of the South Island were excluded from this analysis because of its geographical separation from the rest of the Sub-Antarctic area, the apparent trend that smaller hoki are found there, and the few observed tows in this area. The fishing years 1993–94, 1994–95, and 1997–98 did not have any tows observed in the Puysegur region and a small percentage of the commercial catch was observed in Puysegur during other years. For these reasons, and to be consistent with the current stock assessment where Puysegur catches are combined with west coast South Island catches, the area termed "Sub-Antarctic" will refer to the area south and southwest of the South Island excluding Puysegur (Figure 3).

The Sub-Antarctic observer programme did not collect many samples during July and August. Table 2 shows the median date and the number of weeks in each time period chosen for the Sub-Antarctic region.

The Sub-Antarctic area has three naturally defined areas where fishing takes place: the Campbell Rise (CAMP) to the southeast, the Pukaki (PUKA) Rise on the northeast, and the area in the northwest, referred to as the Plateau (PLAT) (see Figure 3). The first splits from the tree-based regressions typically tried to separate these areas when enough samples were present. Thus, when possible, the samples in each time period were first stratified by these three areas.

For each fishing year, all tows were analysed using tree-based regression to determine likely strata. Common latitude splits occurred near 47.5° S, 49° S, and 51° S. Common longitude splits were near 170° E, and between 167° E and 168° E. Larger fish tended to be found south and/or west and a trend of catching larger fish at deeper net depths has been observed in the catches from the Sub-Antarctic. However, shallow depths show more variation in the mean length per tow (Chris Francis, NIWA, pers. comm.). Net depth was sometimes found as an important stratification variable in this analysis, but some very large fish were observed at shallow depths, which may be midwater tows and/or a result of transcription errors. Nevertheless, in all years, the stratification tended to represent the three areas defined above, through strata defined by depth, latitude, and/or longitude.

O'Driscoll et al. (2002) reported a stratification used for the November-December 2000 Sub-Antarctic trawl survey. They defined 21 strata based on latitude, longitude, and depth, and many of their boundaries are similar to the stratum boundaries found in this analysis.

The final strata within a time period were decided on using the results of the tree-based regression. Two to four strata were common for most time periods, with a few periods having more strata defined when it seemed appropriate. Some variation was explained by the time of day, which was used in 7 of the 48 time periods, mostly in the early years. Fishing method was not used as a splitting variable because there were rarely enough samples using both methods, and depth could usually be used as a proxy for fishing method since midwater gear tended to fish at shallower depths. Other variables may act as proxies for some of the chosen strata, but these possibilities were not investigated closely.

2.2 Mean length at age

Mean length at age data were calculated using data from Chatham Rise and Sub-Antarctic trawl surveys (Table 3). Age data were obtained from Peter Horn at NIWA (unpublished data) and converted into age-length keys using a "catch-at-age" S-plus library incorporating the ideas of Bull & Gilbert (2001). Age-length distributions were then estimated by multiplying length frequency distributions by age-length key matrices, and used to calculate mean lengths at age. Further information can be found in Appendix 1.

The bias and variability of the estimated mean length at age was assessed by bootstrapping. Only the age-length data were resampled, not the length frequency data which are considerably better determined for all but the oldest age groups. Each age-length dataset was resampled 500 times by sampling age-length pairs from the original dataset with replacement. Hence, 500 bootstrapped versions of each set of mean-lengths-at-age were calculated. The bias of each estimate of mean length was estimated as the mean of the corresponding bootstrapped estimates minus the actual estimate, and expressed as a proportion of the actual estimate. The variability of each estimate was estimated as the c.v. of the bootstrapped values.

The bias was negligible, mostly being between –1% and 1% and never exceeding 5%. The c.v.s were small, typically on the order of 1–2%.

3. MODEL FITTING

Predictions of the proportions at age were found using software called Optimised Length Frequency, or OLF, held by the National Institute of Water and Atmospheric Research (NIWA). The model that this software uses is described in Appendix 2. It takes length frequencies and mean lengths at age from specific time periods and estimates cohort specific growth curves with possible year effects as well as the proportion of each cohort in the observed length frequencies. To reduce the number of estimated parameters, sex and stock specific growth curves are estimated which can then be adjusted for certain cohorts by adding in a cohort effect. A cohort effect adjusts the three parameters of the von Bertalanffy growth curve by adding estimated parameters to t_0 and L_∞, and multiplying K by another. Cohort specific variation around the mean length at age is obtained by multiplying an estimated c.v. effect by the overall estimated coefficient of variation. Year effects adjust L_∞ by adding an estimated effect to all cohorts in a particular year. Parameters that account for the length-based selectivity of young fish can also be estimated. These "bias adjustment" parameters shift the true mean lengths at age up to the observed mean lengths at age. The user decides on a maximum age for which these parameters will be estimated, if they are to be included.

This model was first used to estimate the growth parameters and proportions at age for the observer data from the Chatham Rise and Sub-Antarctic areas separately to examine the individual qualities of each area that may be masked in a combined model. Each area was treated as a stock in the model, thus a mixture of stocks was not assumed in the Chatham Rise, but proportions at age for the catch in this area were estimated. The data inputs for each area included the observer length frequencies by sex split up by time periods and mean lengths at age from corresponding trawl surveys in the same area. The observed lengths in each stratum were weighted by the reported TCEPR catch weight in each stratum and combined into a single, sex-specific length frequency using a "catch-at-age" S-plus library held by NIWA. The estimated proportions at age for these two models are not tabulated here because a model with both stocks is to be fitted.

The least squares formulae described in Appendix 2 uses weights on the data types to account for variability and give more weight to reliable data. It was initially decided to give the length frequencies and mean lengths at age respective weights of 10 000 and 1. These values equate a difference of 0.01 in the proportions (fitted in the lfs) with a difference of 1 cm in the mean lengths (i.e. 10 000*$(0.26-0.25)^2$ equals 1*$(56-55)^2$). The length frequencies were arbitrarily down-weighted by 0.25 to 2500 compared to 1 for the mean lengths at age because it is believed that the observer length frequencies have a high amount of variability. This value was chosen because if trawl survey length frequencies were to be used, they would be given a factor of 1 and spawning length frequencies would be given a factor of 0.5. Because the observer length frequencies from these two areas were believed to be less reliable than either of those datasets, they were given the value of 0.25.

As explained in Appendix 2, many different options are available for a model, such as a plus group, bias adjustments, and different effects on growth. Initially, the age of the plus group and the inclusion of bias adjustments were investigated by looking at fits to the data without any effects added. The results of this investigation were used in all of the models. For each model, cohort, c.v., and year effects were tested to determine the combination that would result in the best fit, as indicated by the final objective value. Single effects were introduced to the model one at a time and the one that reduced the objective value the most was used. This process was repeated until no effects decreased the objective value by more than 1%. Fitted length frequency and standardised residual plots were looked at to confirm that the newly added effect was improving the model in an appropriate way. Seasonal effects were tested after most cohort and year effects were added.

The two areas were then combined into a "combined" model where the Chatham Rise was considered to contain a mixture of the two hoki stocks. When calculating the mean length at age for a length frequency where the two stocks are mixed, it was assumed that the sample contains 50% of each stock. Significant effects for the combined model were chosen using the stepwise procedure stated above.

After a final model was chosen, the predicted proportions at age and sex in the commercial catch over the entire fishing year were calculated using the formulae described in Appendix 3. This involves weighting the proportions at age for each sex and time period by the proportion of each sex in the observed length frequency and total catch from the fishery in each time period of the fishing year. These results were then compared to results from MIX analyses that have been used in previous stock assessments.

Standardised residuals were used to assess the fit of the models and to determine if the least squares approach is valid and the weightings are correct. A standardised residual was calculated as

$$\hat{s}_j = \sqrt{\frac{w_j(O_{ij}-P_{ij})^2}{\frac{\sum_{j'}\sum_{i'} w_{j'}(O_{i'j'}-P_{i'j'})^2}{n}}}$$

where:

O_{ij} is the ith observation of data type j,
P_{ij} is the ith prediction of data type j,
w_j is the weight used for dataset j, and
n is the total number of observations (proportions and mean length at age).

Heteroskedasticity was looked for in plots of the standardised residuals to indicate if the least squares may be a valid approach. Also, the standard deviation of the standardised residuals for an individual data type *j* (length frequencies or mean length at age) should be near 1 if the weightings explained earlier are not wildly wrong.

There is some concern that the stratification of the observer data may have caused non-representative length frequencies because of the small number of tows used in some strata. However, few tows are available, especially after splitting the year into time periods (see Tables 1 and 2). If we were to split each time period into strata based on the splitting criteria specified by Francis (2002), the observer data would be rarely stratified any further than these time periods. Therefore, a sensitivity case, using the combined model, was run using length frequencies that were not stratified within each time period.

4. RESULTS

It was found that a plus group at age 10 was needed to accurately estimate the right side of the length distribution. Without a plus group, the longer lengths were underestimated because there was no model for large fish. When the age of the plus group was less than 10, the distribution of the plus group did not fully reach the right tail of the length frequency distribution. Age 10 was chosen because there was still some separation between estimated mean lengths at age and the right end of most estimated length frequency distributions matched well with the respective observed distributions.

A bias correction for ages 1 and 2 showed a significant decrease in the objective value for both areas and tended to fit the modes at short lengths in the length frequency distributions much better. A bias correction for age 3 did not show much improvement, but it was noticed that the mean length at age 3 was shifted to slightly higher values when the bias correction was included for ages 1 and 2, compared to no bias correction. The mean length at age 2 for the Sub-Antarctic data tended to overestimate the observed data in some years, but adding in a cohort effect for the 1997 cohort alleviated the problem. The bias correction up to age 2 was used for all subsequent runs.

4.1 Chatham Rise

Cohort, c.v., and year effects were significant in the final Chatham Rise model. A 1997 cohort effect and 1984 c.v. effect were important, and year effects for 1987, 1988, 1997, 1998, and 1999 significantly reduced the sum of squares. The model with no effects had a final objective value of 3811, which was reduced to 3176 with all of the above effects. Seasonal effects made no improvements to this model, thus were not included. The values of all the estimated parameters are listed in Table 4.

The cohort effect caused the 1997 cohort to grow faster, but to a slightly lesser maximum length. The parameter t_0 increased by 0.17, K increased by a factor of 1.17, and L_∞ decreased slightly by 0.3 cm. The year effects decreased L_∞, except for 1998, which increased L_∞ by 5.7 cm from values of 82.1 and 87.0 for males and females, respectively. The 1987 and 1988 years showed a slight decrease in the maximum length of about 0.7 cm, while 1997 and 1999 showed a large decrease of 3.0 and 2.3 cm, respectively, although the decrease in 1999 is correcting for the increase in 1998. The c.v. effect in 1984 was estimated at its lower bound of 0.2, but was not changed and re-estimated because a combined model was to be run. Nevertheless, the model wanted the 1984 cohort to have little variation in length, possibly even trying to eliminate that cohort altogether.

The standard deviations of the standardised length frequency residuals and mean length at age residuals were 0.78 and 3.99, respectively. This suggests that the weighting may be too low for the length frequency data. The fits to the observed length frequencies of the 1997 cohort at age 2 are slightly off, as seen in Figure 4. Similar plots of other time periods (when the 1997 cohort was not present or not 2 years old) did not show any severe lack of fit.

A high proportion of the 1991, 1992, and 1994 cohorts were observed in the catch during the years when they were of the ages 1 to 4. Other cohorts represented with high proportions in the catch were 1987 and 1997.

4.2 Sub-Antarctic

The model for the Sub-Antarctic area showed significant cohort effects for the 1988 and 1989 cohorts, and c.v. effects for 1988 and 1997. The objective value for the model with no effects was 4031, being reduced to 3443 by adding all of the effects. The estimated parameters are tabulated in Table 5.

The cohort effects for 1988 and 1989 made these cohorts faster growing than all the others by adding to all of the growth parameters. The parameter L_∞ increased by 1.08 and 1.59, t_0 increased by 0.14 and 0.11, and K increased by a factor of 1.10 and 1.06 for the 1988 and 1989 cohorts, respectively. Both c.v. effects were estimated at their lower bound of 0.2, but were not investigated further since a combined model would be fitted where c.v. effects would be studied if they appeared. There are no year effects that improved the fit.

As with the Chatham Rise model, there was a lack of fit to the 1997 cohort when it was age 2. One reason for this may be the large mean lengths at age observed for the 1997, 1996, and 1995 cohorts from the December 2000 Sub-Antarctic survey (ages 3, 4, and 5). The discrepancy between the lengths observed from the survey and the predicted lengths from the model can be seen in Figure 5.

The model estimated an unusually high proportion of the 1991 cohort as 1+ year olds during the 1992–93 fishing year. The time periods in February and April 1993 had a large number of fish near a length of 40 cm in the commercial catch and the May-June 1993 Sub-Antarctic survey by *Tangaroa* showed a similar trend (Ballara et al. 2000). However, in subsequent years, a high proportion of the 1991 cohort was not observed in these results.

The standard deviations of the length frequency residuals and mean length at age residuals were 0.93 and 3.47, respectively. This suggests that the weighting may be too high for the mean length at age data, although the small number of mean length at age residuals may lead to an unreliable estimate of the standard deviation of the standardised residuals.

Of concern in this model is that the proportions of the 1987 and 1988 cohorts appeared to be poorly estimated. The 1987 cohort was consistently estimated with a high proportion because the c.v. effect for the 1988 cohort made the 1988 cohort almost non-existent. These two cohorts have an estimated mean length at age that is very close, due to the cohort effect for 1988, and, unfortunately, it appears that the model has a difficult time discriminating between the two cohorts in the catch sampled, giving most of the proportion to the 1987 cohort with a wider distribution. Figure 6 shows the predicted and observed mean lengths at age from the December 1993 trawl survey when the 1988 and 1987 cohorts would have been ages 5 and 6, respectively. The predicted mean lengths at age for the two cohorts are very similar, although the mean length for the 1988 is under-predicted. Removing the 1988 c.v. effect caused the 1988 cohort to have a higher estimated proportion of the catch in the fishing years 1993–94 onward.

4.3 Combined model

The final combined model included a 1997 cohort effect for both stocks, agreeing with the Chatham Rise model, and caused the 1997 cohort to grow faster than other cohorts (Figure 7). Year effects for 1991, 1997, and 1998 adjusted L_∞ by 1.82, 3.54, and –1.01, respectively. No c.v. effects significantly improved the fit. The method bias showed a slightly negative parameter for the age 1 fish from the Sub-Antarctic, which should not occur if the bias parameter is a proxy for a shift in the mean length at age due to selectivity. The estimated growth parameters are listed in Table 6. The standard deviations of the standardised length frequency residuals and mean length at age residuals showed similar results to the individual area models, suggesting the mean length at age data were weighted too high.

The model results fit the observer length frequencies from both the Chatham Rise and Sub-Antarctic very well (Figures 8 and 9). The residuals plotted against length show no trends for the Chatham Rise in Figure 10. However, a worse fit is seen in the Sub-Antarctic data and some trends in the residuals show that predicted nodes in the length frequencies were observed as a spike (i.e., curved residual trend below the zero line). These few spiky observed length frequencies may represent inadequate or biased sampling, and increasing the minimum number of tows that one time period must contain may help alleviate this problem. The mean lengths at age from the trawl surveys show some discrepancies, mostly with young and old ages from early surveys, where there appears to be a lot of variability (Figures 11–14). The Chatham Rise residuals from the mean length at age data (Figure 12) over-predict age 5 for all but one survey. The Chatham Rise mean lengths for ages 4 and 6 also tend to be over-predicted. The Sub-Antarctic mean length at age data show a good pattern in the residuals with no age class consistently over-predicted or under-predicted. However, the residuals tend to be slightly larger for the young age classes.

The predicted proportions of each cohort in the catch for the fishing years 1991–92 to 1999–2000 were mostly similar to the predicted proportions obtained from the individual area models, as can be seen in Figures 15 and 16. However, because a c.v. effect was not included for the 1988 cohort, the model had difficulty discriminating between the 1987 and 1988 cohorts. The 1987 cohort was consistently estimated at high proportions except in the 1993–94 fishing year when the trend is reversed (Figure 16). This phenomenon was not observed in the Sub-Antarctic model because of the 1988 c.v. effect. Since c.v. effects cannot be stock specific, it did not appear here. A 1988 c.v. effect was introduced into the combined model reducing the c.v. for that cohort only by a factor of 0.7, probably because of the influence from the Chatham Rise data. There was little difference in the results with and without this c.v. effect. It may be that the c.v. effect should be stock specific, which is currently not possible to model in this version of OLF, or the c.v. effect does not behave in the manner expected and acts to eliminate some cohorts.

The predicted proportions from the combined model for each cohort in each fishing year and area are given in Tables 7 and 8. The birth date of hoki in this model was 1 July, thus a cohort that is age 0+ before that date become age 1+ for a short time during the end of the fishing year (July–September). Since the young cohort was not estimated in the model previous to 1 July, their proportion is likely biased low. However, it is unlikely that they appear in any significant numbers in the commercial catch.

A consistently high proportion of the catch consisted of the 1987, 1988, 1991, 1992, 1994, and 1997 cohorts, suggesting that they are strong year classes for both stocks (Figures 17 and 18), although it is difficult to determine the strength of the eastern stock because the only data in this model for that stock contain a mixture of the two. The strong cohorts from the Chatham Rise are consistent between the two sexes and across fishing years, but the Sub-Antarctic shows variable proportions in the catch among the fishing years. Most noticeable is the sudden low proportion of the male 1987 year class in the 1993–94 fishing year.

The proportions of each age and sex in the commercial catch from the Chatham Rise that were used in previous stock assessments are shown in Table 9. These proportions are only for ages 2–6, where age 6 is a plus group. A comparison between these proportions from previous MIX analyses and the predicted proportions from the combined model are shown in Figure 19. The early years show some differences, but in recent years the proportions are more similar.

The predicted proportions for each cohort and sex that were used in previous stock assessments indicate that the 1987 and 1988 cohorts made up a large proportion of the Sub-Antarctic catch over the 1992–93 fishing year (Table 9). The females caught in the Sub-Antarctic show similar results in the combined model, but the proportions of the males predicted here do not entirely agree (Figure 20). In this analysis the strength of the Sub-Antarctic males from the 1988 cohort is unknown, and may be due to lack of data for the years that this cohort was present in the catch (ages 5–7 or fishing years 1993–94 through 1995–96). The previous results and the current results from the Chatham Rise show high proportions of both cohorts, although the MIX analysis suggests a larger proportion of the 1988 cohort.

A slight discrepancy also occurs for cohorts in the early 1990s. The combined model typically estimated higher proportions of the 1992 cohort and less of the 1991 cohort than the previous MIX analysis, and the current model does not show the 1993 year class as strong as the MIX results did, especially in 1996–97. However, the two methods appear to have similar results in the most recent years.

Using otolith ages is a promising method of calculating catch at age and has been used on the Chatham Rise for the 1998–99 and 1999–2000 fishing years (Francis 2002). A comparison of the year class frequencies estimated by Francis (2002) using "with scores" ages and stratification to the proportions at age and sex estimated in the combined model is shown in Figure 21. The 1998–99 fishing year shows similar results between the two methods, although the combined model shows a larger difference between the 1993 and 1994 cohorts than the otolith ages show. The estimated proportions from both methods are very similar in the 1999–2000 fishing year.

Parameters for the combined model were also estimated using time periods that were not stratified. The growth and bias parameters resulted in nearly the same estimates as when stratifying each time period (Table 10). This suggests that there is no spatial variability in growth. A comparison between the estimated proportions at age and sex in Figures 22 and 23 show that the proportions are very similar between the models for both areas. There is more variability in the Sub-Antarctic estimated proportions, but the same trends are still present, especially the showing of a strong 1988 cohort in 1993–94.

To help determine if the least squares approach was reasonable for estimating the proportions at length, the absolute standardised residuals were plotted. Figure 24 shows the residuals plotted against the predicted proportion with a loess line drawn through them to help determine if any trends are present, indicating heteroskedasticity. The increasing line suggests that the variance increases with the proportion and least squares may not be appropriate.

5. CONCLUSION

The similarities between the single area models and the combined model show that this method can be used to estimate the proportion at age and sex for the Chatham Rise and Sub-Antarctic commercial catches either individually or together in one model. Estimating Chatham Rise proportions alone treats that area as a single stock without assuming the proportion of each stock in the area, which may be desirable because the proportion of each stock changes with age as the western stock migrates to its home ground, changes from year to year, and it is difficult to estimate the proportion of each stock in a sample (Hicks & Gilbert 2002). The estimated growth parameters for the Chatham Rise model showed a curve between that estimated for each stock in the combined model. The proportions at age and sex were very similar between the two models (Figure 15). The assumption that the stocks mix on the Chatham Rise in an even ratio may not be reasonable, resulting in an inaccurate mean length at age estimated in the model.

Other data that may be used to fit the growth curves are the spawning area length frequencies or estimated spawning area mean lengths at age. The spawning populations have been well sampled since 1988 (Ballara & Livingston 2001) and may contain some stock specific length and year class strength information. However, in this analysis, we felt that the spawning population may have a different age-length relationship, especially at young ages where faster maturing fish may grow faster, and thus did not include that information.

The 1987 and 1988 cohorts in the Sub-Antarctic showed inconclusive results and stress the need for the continued collection of quality data to help estimate proportions at age in the catch. The proportions of both cohorts are fairly large in the Chatham Rise data, and the females from the 1986–1988 cohorts show large proportions in the Sub-Antarctic catch. The males of both cohorts are likely present in the commercial catch, but given the data, are not easily separated. The Sub-Antarctic model did not show the inconsistency, but it also included a 1988 c.v. effect, which virtually eliminated the 1988 cohort by *shrinking the variability around the mean length at age.* The amount of observer data collected from the Sub-Antarctic in the early 1990s was less than that collected on the Chatham Rise and may not have been enough to accurately predict the proportions of each cohort and sex.

The model appeared to predict the proportions of each cohort in the commercial catch fairly well, and using a plus group at age 10 results in more output than the previous MIX analyses with a plus group at age 6. However, attempting to predict proportions at age for hoki over 6 years old may prove difficult because the lengths considerably overlap at these older ages. A plus group at age 10 was chosen in the model because the predicted length frequency fits the right tail of the observed length frequencies better than when using younger plus groups, but the final proportions at age and sex can be easily collapsed into a younger plus group. When pooling the predicted proportions into a plus group at age 6, similar results are seen to those predicted by previous MIX analyses, and year classes with a high proportion in the catch are in agreement with those presented by Ballara & Livingston (2001).

A good way to determine how well this model predicts year class proportions in the commercial catch is to compare the results with proportions determined from aged otoliths collected from the commercial catch. The predicted proportions reported by Francis (2002) using otolith ages agree with the results from the combined model for the Chatham Rise area, although the otolith method seems to be more evenly spread out, which could be due to ageing error. A comparison between the otolith ages and the results from MIX analyses (Ballara et al. 2002) showed similar differences to the combined model. However, the MIX analyses did not show as large a difference for the 1994 cohort in the 1998–99 fishing year.

Of considerable concern is how much the stratification within each time period affects the model. As was seen when comparing the combined models with and without spatial stratification, very little difference occurred. It is likely that with the small number of samples present in each time period, the temporal stratification was sufficient. This leads to the issue of how temporal stratification should be dealt with. In this analysis, the time periods were chosen by aggregating each fishing year into clusters of 4 to 12 weeks, which sometimes created periods with few samples in them because little sampling was done over a long period, or it was the start or end of the fishing year. In years with good observer coverage, temporal stratification will be simple and can possibly be standardised for every year, or the tree-based regression can be used with only date as an input to try and explain the variability in mean lengths over the year. However, in the past, sampling has been sporadic and the choice of time periods may have an effect on the estimated proportions at age. That effect was not determined in this analysis.

An aspect of the model that can use improvement is how different years affect growth. First, the parameterisation of the year effects is not intuitive. Even though a year effect may be estimated as negative, it does not mean the cohort is experiencing reduced growth when compared to average. Instead, the model may be compensating for large year effects in previous years because a year effect is always added to L_∞ once that year has passed. For example, the 1998 and 1999 year effects in the combined model are 2.94 and –0.61, respectively. At the end of 1998, a cohort will have 2.94 added onto L_∞. Then, in 1999 that cohort will have 0.61 subtracted from L_∞, but after the 1998 year effect has been added. Therefore, even though the 1999 year effect is negative, the resulting effect added onto L_∞ for a cohort in 1999 is 2.33 more than in 1997. More importantly, the year effect may not be an adequate depiction of yearly growth patterns because a simple change in L_∞ affects older fish more than younger fish. In reality, the opposite would seem true since growth slows as a fish grows old. Finally, a year effect means that a cohort experiences a new growth curve that year, thus in a bad year with an overall negative effect, the mean length of older fish will decrease making them smaller, which is not realistic. It may be better to introduce year effects by modelling growth increments rather than adjusting a parameter from the von Bertalanffy growth equation.

The use of weights in the least squares formulation is one way to deal with the variability of different data sets, but may not adequately deal with the variability within a dataset, unless a specific weight is chosen for each length frequency. The analysis of the standardised residuals to determine if the weights were appropriate showed that the mean lengths at age may be weighted too high. However, the mean length at age data did not fit very well and a pattern was seen in the residuals plotted against age for the Chatham Rise data. Increasing the weights for the mean length at age data may force the model to fit that data better, thus defining the position of the mean length at age distributions in the length frequencies better. This contradiction may be a result of using the least squares approach instead of likelihood methods.

Also, the standardised residuals plotted against the predicted proportion of each length bin (Figure 24) show an increasing trend of variance with fitted values. This heteroskedasticity indicates that the use of least squares may not be adequately modelling the variance structure, and may be more adequately modelled using a multinomial type formulation. A robust likelihood for estimating proportions at age was developed by Fournier et al. (1990) that deals with the increasing variance with proportion present in multinomial data and may be more appropriate.

Overall, the OLF software is easy to use and the model has the flexibility to incorporate cohort and year effects on growth. The cohort specific c.v. effects seemed to try to eliminate certain cohorts from the final estimated proportions at age, and may not be an adequate method of modelling cohort specific variation in lengths at age. Some improvements can be made to the model, but the estimates of proportions at age and sex seem reasonable and are an improvement over the previous MIX analysis because growth over the sampling year is incorporated, multiple data sources can be used to help determine the proportions at age, the estimates are obtained by using available data over a number of years rather than using length frequencies from only the year of interest, and the subjectivity of defining the mean length at age, as in MIX analyses, is not present in this model.

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7. REFERENCES

Annala, J.H.; Sullivan, K.J.; O'Brien, C.J. (comps) (2001). Report from the fishery assessment plenary, May 2001: stock assessments and yield estimates. (Unpublished report held in NIWA library, Wellington).

Ballara, S.L.; Cordue, P.L.; Livingston, M.E. (2000). A review of the 1997–98 hoki fishery and assessment of hoki stock for 1999. *New Zealand Fisheries Assessment Report 2000/8.* 65 p.

Ballara, S.L.; Cordue, P.L.; Livingston, M.E.; O'Driscoll, R.L.; Kendrick, T.H. (2002). A review of the 1999–2000 hoki fishery and assessment of the hoki stocks for 2001. *New Zealand Fisheries Assessment Report 2002/4.* 87 p.

Ballara, S.L.; Livingston, M.E. (2001). Catches, size, and age structure of the 1998–99 hoki fishery. *New Zealand Fisheries Assessment Report 2001/6.* 63 p.

Bradford, E. (2000). How representative is the sampling by the observer programme of the commercial hoki catches on the Chatham Rise? Final Research Report for Ministry of Fisheries Research Project HOK1999/04 Objective 2 (in part). 20 p. (Unpublished report held by the Ministry of Fisheries, Wellington.)

Bull, B.; Gilbert, D.J. (2001). Catch-at-age sampling. *New Zealand Fisheries Assessment Report 2001/53.* 19 p.

Fournier, D.A.; Sibert, J.R.; Majkowski, J.; Hampton, J. (1990). MULTIFAN: a likelihood-based method for estimating growth parameters and age composition from multiple length frequency data sets illustrated using data for southern bluefin tuna (*Thunnus maccoyii*). *Canadian Journal of Fisheries and Aquatic Sciences 47*: 301–317.

Francis, R.I.C.C. (2002). Estimating catch at age in the Chatham Rise hoki fishery. *New Zealand Fisheries Assessment Report 2002/9.* 20 p.

Hicks, A.C.; Gilbert, D.J.; (2002). Stock discrimination of hoki (*Macruronus novaezelandiae*) based on otolith ring measurements. *New Zealand Fisheries Assessment Report 2002/2.* 31 p.

Land Information New Zealand. (2001). Sunrise and sunset tables 2001. http://www.hydro.linz.govt.nz/astro/sun2001.pdf

O'Driscoll, R.L.; Bagley, N.W.; Bull, B. (2002). Trawl survey of middle depth species in the Southland and Sub-Antarctic areas, November-December 2000 (TAN0012). *NIWA Technical Report* 110. 78 p.

Table 1: Time periods for the Chatham Rise area in which the length frequencies were calculated along with the number of weeks, number of observed tows where fish were sexed, and the number of strata in each time period. The variables that were used to split the strata are also listed with the first chosen on top. The date is the median date of the time period and some weeks were excluded due to periods of no sampling.

Year		Time period							
		1	2	3	4	5	6	7	8
1991–92	Date	9 Dec	10 Feb	20 Apr	3 Jul	5 Sep			
	# weeks	8	10	10	11	7			
	# tows	51	38	74	14	8			
	# strata	2	3	5	2	1			
	splitting variables	depth	depth longitude	longitude depth	depth				
1992–93	Date	28 Oct	23 Dec	21 Apr	7 Jul	16 Sep			
	# weeks	8	8	12	10	4			
	# tows	5	43	16	11	14			
	# strata	1	2	2	2	2			
	splitting variables		longitude	depth	depth	longitude			
1993–94	Date	28 Oct	16 Dec	20 Jan	21 Feb	18 Apr	9 Jun		
	# weeks	8	6	4	5	7	8		
	# tows	66	57	39	31	35	55		
	# strata	3	4	4	2	3	3		
	splitting variables	depth latitude	depth longitude	longitude latitude	depth	longitude depth	depth longitude		
1994–95	Date	11 Oct	25 Nov	14 Feb	9 May	7 Jul	9 Sep		
	# weeks	3	10	13	11	6	6		
	# tows	6	13	30	71	22	16		
	# strata	1	2	2	5	2	3		
	splitting variables		depth	longitude	depth latitude night	depth	depth longitude		
1995–96	Date	14 Oct	30 Dec	1 May	22 Jun	15 Sep			
	# weeks	4	12	7	8	4			
	# tows	49	56	70	57	66			
	# strata	5	4	5	2	3			
	splitting variables	depth longitude	depth longitude night	latitude longitude depth	longitude depth	latitude longitude			
1996–97	Date	18 Oct	25 Apr	13 Jun	28 Jul				
	# weeks	5	7	7	6				
	# tows	70	48	41	12				
	# strata	6	3	4	2				
	splitting variables	longitude latitude depth	longitude depth	latitude longitude depth	latitude				
1997–98	Date	28 Oct	9 Dec	6 Jan	3 Feb	7 Mar	14 Apr	13 Jun	9 Sep
	# weeks	8	4	4	4	5	6	11	6
	# tows	32	79	60	43	78	274	133	16
	# strata	3	5	6	6	4	4	3	2
	splitting variables	longitude	longitude	longitude depth	depth longitude	depth longitude	longitude depth	longitude depth	longitude

Table 1: ***continued***

Year									Time Period
		1	2	3	4	5	6	7	8
1998–99	Date	28 Oct	9 Dec	6 Jan	3 Feb	3 Mar	7 Apr	19 May	4 Jul
	# weeks	8	4	4	4	4	6	6	7
	# tows	98	53	91	72	24	45	112	44
	# strata	3	4	5	5	3	3	4	4
	splitting variables	longitude depth	depth latitude longitude	depth longitude	longitude latitude	depth latitude	longitude	latitude depth longitude	latitude depth longitude
1999–00	Date	21 Oct	2 Dec	31 Jan	27 Apr	5 Jun	19 Sep		
	# weeks	6	6	11	6	5	3		
	# tows	42	92	23	88	109	26		
	# strata	2	6	3	4	5	3		
	splitting variables	depth	latitude longitude night depth	latitude depth	depth longitude	latitude depth longitude	longitude latitude		

Table 2: Time periods for the Sub-Antarctic area in which the length frequencies were calculated along with the number of weeks, number of observed tows where fish were sexed, and the number of strata in each time period. The variables that were used to split the strata are also listed with the first chosen on top. The date is the median date of the time period and some weeks were excluded due to periods of no sampling.

Year									Time Period
		1	2	3	4	5	6	7	8
1991–92	Date	14 Oct	11 Nov	24 Jan	10 Apr	25 May	17 Jul	5 Sep	
	# weeks	4	4	5	5	8	7	7	
	# tows	65	31	18	24	59	35	34	
	# strata	4	4	2	4	4	3	4	
	splitting variables	latitude depth	longitude depth night	depth	depth night longitude	longitude depth	depth latitude	latitude longitude	
1992–93	Date	18 Oct	20 Dec	24 Feb	14 Apr	26 Aug			
	# weeks	5	7	6	8	10			
	# tows	56	16	22	42	49			
	# strata	4	2	3	5	4			
	splitting variables	longitude depth	longitude	latitude night	depth night longitude	latitude longitude			
1993–94	Date	21 Oct	29 Nov	7 Mar	16 Sep				
	# weeks	6	5	9	4				
	# tows	32	32	30	6				
	# strata	5	4	4	1				
	splitting variables	latitude longitude depth	latitude depth	latitude night					
1994–95	Date	17 Jan	21 Feb	25 Apr					
	# weeks	5	5	7					
	# tows	35	30	14					
	# strata	3	4	2					
	splitting variables	longitude night	latitude depth	latitude					

Table 2: ***continued***

Year		Time Period							
		1	2	3	4	5	6	7	8
1995–96	Date	11 Oct	24 Feb	4 May	15 Sep				
	# weeks	3	8	12	4				
	# tows	18	22	59	15				
	# strata	3	3	6	3				
	splitting variables	latitude longitude	latitude depth	latitude longitude depth	latitude longitude				
1996–97	Date	14 Oct	22 Nov	27 Jan	31 Mar	16 Sep			
	# weeks	4	7	4	8	4			
	# tows	14	7	5	18	18			
	# strata	2	1	1	3	3			
	splitting variables	depth			latitude longitude	latitude depth			
1997–98	Date	22 Nov	27 Jan	3 Mar	14 Apr	16 Sep			
	# weeks	7	4	6	6	4			
	# tows	35	30	30	38	41			
	# strata	2	3	3	4	3			
	splitting variables	latitude	latitude	depth	latitude	latitude longitude			
1998–99	Date	14 Oct	10 Feb	14 Mar	14 Apr	19 May	23 Jun	16 Sep	
	# weeks	4	4	5	4	6	4	4	
	# tows	14	52	35	54	106	19	22	
	# strata	2	4	5	4	8	2	3	
	splitting variables	latitude	latitude depth	latitude longitude	depth longitude latitude	depth longitude night latitude	longitude	longitude latitude	
1999–00	Date	14 Oct	13 Jan	10 Feb	13 Mar	13 Apr	18 May	26 Jun	15 Sep
	# weeks	4	4	4	5	4	6	5	4
	# tows	31	39	56	50	49	160	74	19
	# strata	3	3	5	5	4	5	3	2
	splitting variables	latitude longitude	latitude	latitude longitude depth	latitude longitude	depth longitude latitude	latitude longitude depth	latitude depth	longitude

Table 3: Median dates of the surveys used to calculate mean length at age.

Chatham Rise		Sub-Antarctic	
Survey code	Median date	Survey code	Median date
TAN9106	14 Jan 1992	TAN9105	3 Dec 1991
TAN9212	17 Jan 1993	TAN9204	4 May 1992
TAN9401	17 Jan 1994	TAN9209	30 Sep 1992
TAN9501	15 Jan 1995	TAN9211	3 Dec 1992
TAN9601	5 Jan 1996	TAN9304	18 May 1993
TAN9701	13 Jan 1997	TAN9310	2 Dec 1993
TAN9801	13 Jan 1998	TAN9605	12 Apr 1996
TAN9901	15 Jan 1999	TAN9805	19 Apr 1998
TAN0001	9 Jan 2000	TAN0012	8 Dec 2000
TAN0101	11 Jan 2001		

Table 4: Parameter estimates from the Chatham Rise model.

	Growth parameters		Year effects		Method bias	
	Male	Female				
K	0.34	0.32	1987	-0.76	bias age 1	2.64
t_0	-0.23	-0.23	1988	-0.71	c.v. age 1	1.27.
L_∞	82.10	86.99	1997	-3.06	bias age 2	1.30
$C_{K,1997}$	1.17		1998	5.74	c.v. age 2	1.09
$C_{t_0,1997}$	0.17		1999	-2.31		
$C_{L_\infty,1997}$	-0.28					
c.v.	0.043					
$C_{cv,1984}$	0.2					

Table 5: Parameter estimates from the Sub-Antarctic model.

	Growth parameters		Method bias	
	Male	Female		
K	0.2968	0.2579	bias age 1	0.83
t_0	-0.2683	-0.3354	c.v. age 1	0.74
L_∞	89.2535	96.4613	bias age 2	3.68
$C_{K,1988}$	1.0992		c.v. age 2	0.64
$C_{t_0,1988}$	0.1406			
$C_{L_\infty,1988}$	1.0812			
$C_{K,1989}$	1.0646			
$C_{t_0,1989}$	0.1093			
$C_{L_\infty,1989}$	1.5886			
c.v.	0.0432			
$C_{cv,1988}$	0.2			
$C_{cv,1997}$	0.2			

Table 6: Parameter estimates from the combined model.

	Growth parameters				Year effects		Method bias	
	Eastern stock		Western stock					
	Male	Female	Male	Female				
K	0.47	0.48	0.32	0.28	1991	1.80	CHAT bias age 1	2.29
t_0	-0.091	-0.024	-0.23	-0.28	1998	2.94	CHAT c.v. age 1	1.27
L_∞	70.25	72.27	87.20	92.93	1999	-0.61	CHAT bias age 2	0.76
$C_{K,1997}$	1.21		1.10				CHAT c.v. age 2	0.98
$C_{t_0,1997}$	0.17		0.11				SUBA bias age 1	-0.25
$C_{L_\infty,1997}$	-0.22		-0.20				SUBA c.v. age 1	0.69
c.v.	0.0469						SUBA bias age 2	2.36
							SUBA c.v. age 2	1.44

Table 7: Estimated proportions of each cohort and sex in the commercial catch from the Chatham Rise for each fishing year from 1991–92 to 1999–2000 using the combined model. The oldest cohort is a plus group and the youngest would be age 1 near the end of the fishing year, being estimated only during the latter part of the season.

MALES

Year	1999	1998	1997	1996	1995	1994	1993	1992	1991	1990	1989	1988	1987	1986	1985	1984	1983	1982	1981
1991–92	—	—	—	—	—	—	—	—	0.000	0.001	0.003	0.146	0.213	0.017	0.008	0.007	0.007	0.007	0.001
1992–93	—	—	—	—	—	—	—	0.000	0.068	0.006	0.024	0.136	0.109	0.014	0.008	0.002	0.003	0.040	—
1993–94	—	—	—	—	—	—	0.000	0.057	0.130	0.007	0.023	0.050	0.069	0.055	0.003	0.010	0.007	—	—
1994–95	—	—	—	—	—	0.000	0.015	0.167	0.086	0.041	0.005	0.049	0.031	0.003	0.002	0.014	—	—	—
1995–96	—	—	—	—	0.000	0.073	0.065	0.183	0.063	0.033	0.008	0.001	0.002	0.024	0.005	—	—	—	—
1996–97	—	—	—	0.000	0.006	0.229	0.027	0.147	0.018	0.021	0.008	0.003	0.003	0.008	—	—	—	—	—
1997–98	—	—	0.000	0.016	0.025	0.252	0.047	0.036	0.028	0.006	0.007	0.005	0.011	—	—	—	—	—	—
1998–99	—	0.000	0.051	0.044	0.091	0.178	0.020	0.018	0.017	0.006	0.004	0.004	—	—	—	—	—	—	—
1999–00	0.000	0.035	0.155	0.044	0.060	0.028	0.025	0.002	0.002	0.018	0.024	—	—	—	—	—	—	—	—

FEMALES

Year	1999	1998	1997	1996	1995	1994	1993	1992	1991	1990	1989	1988	1987	1986	1985	1984	1983	1982	1981
1991–92	—	—	—	—	—	—	—	—	0.000	0.001	0.004	0.202	0.263	0.038	0.005	0.022	0.023	0.020	0.009
1992–93	—	—	—	—	—	—	—	0.000	0.088	0.005	0.043	0.169	0.188	0.000	0.007	0.002	0.041	0.048	—
1993–94	—	—	—	—	—	—	0.000	0.054	0.154	0.013	0.016	0.066	0.165	0.072	0.003	0.014	0.031	—	—
1994–95	—	—	—	—	—	0.000	0.015	0.164	0.097	0.052	0.011	0.062	0.053	0.013	0.032	0.087	—	—	—
1995–96	—	—	—	—	0.000	0.072	0.075	0.186	0.080	0.023	0.014	0.006	0.026	0.040	0.020	—	—	—	—
1996–97	—	—	—	0.000	0.009	0.218	0.049	0.160	0.023	0.013	0.010	0.013	0.014	0.019	—	—	—	—	—
1997–98	—	—	0.000	0.019	0.031	0.282	0.060	0.066	0.048	0.008	0.009	0.015	0.029	—	—	—	—	—	—
1998–99	—	0.000	0.056	0.041	0.093	0.243	0.018	0.054	0.022	0.004	0.015	0.021	—	—	—	—	—	—	—
1999–00	0.000	0.029	0.171	0.055	0.076	0.044	0.037	0.003	0.012	0.049	0.131	—	—	—	—	—	—	—	—

Table 8: Estimated proportions of each cohort and sex in the commercial catch from the Sub-Antarctic for each fishing year from 1991–92 to 1999–2000 using the combined model. The oldest cohort is a plus group and the youngest would be age 1 near the end of the fishing year, being estimated only during the latter part of the season.

MALES

Year	1999	1998	1997	1996	1995	1994	1993	1992	1991	1990	1989	1988	1987	1986	1985	1984	1983	1982	1981
1991–92	—	—	—	—	—	—	—	—	0.000	0.000	0.000	0.038	0.196	0.046	0.026	0.039	0.013	0.016	0.017
1992–93	—	—	—	—	—	—	—	0.000	0.117	0.003	0.004	0.039	0.116	0.026	0.028	0.006	0.011	0.014	—
1993–94	—	—	—	—	—	—	0.000	0.003	0.029	0.012	0.001	0.234	0.041	0.085	0.084	0.005	0.011	—	—
1994–95	—	—	—	—	—	0.000	0.015	0.092	0.078	0.043	0.008	0.019	0.120	0.003	0.019	0.005	—	—	—
1995–96	—	—	—	—	0.000	0.002	0.038	0.217	0.151	0.022	0.047	0.020	0.015	0.020	0.014	—	—	—	—
1996–97	—	—	—	0.000	0.000	0.016	0.072	0.255	0.020	0.032	0.013	0.003	0.007	0.080	—	—	—	—	—
1997–98	—	—	0.000	0.004	0.021	0.140	0.058	0.074	0.056	0.027	0.012	0.021	0.016	—	—	—	—	—	—
1998–99	—	0.000	0.061	0.021	0.085	0.147	0.023	0.070	0.051	0.005	0.012	0.012	—	—	—	—	—	—	—
1999–00	0.000	0.001	0.075	0.009	0.074	0.095	0.056	0.031	0.070	0.012	0.003	—	—	—	—	—	—	—	—

FEMALES

Year	1999	1998	1997	1996	1995	1994	1993	1992	1991	1990	1989	1988	1987	1986	1985	1984	1983	1982	1981
1991–92	—	—	—	—	—	—	—	—	0.000	0.000	0.000	0.035	0.159	0.114	0.021	0.042	0.106	0.061	0.069
1992–93	—	—	—	—	—	—	—	0.000	0.352	0.002	0.001	0.031	0.120	0.023	0.017	0.051	0.019	0.020	—
1993–94	—	—	—	—	—	—	0.000	0.003	0.041	0.023	0.014	0.128	0.092	0.086	0.017	0.061	0.029	—	—
1994–95	—	—	—	—	—	0.000	0.010	0.107	0.061	0.057	0.000	0.106	0.089	0.065	0.065	0.038	—	—	—
1995–96	—	—	—	—	0.000	0.002	0.055	0.099	0.126	0.015	0.037	0.009	0.009	0.083	0.022	—	—	—	—
1996–97	—	—	—	0.000	0.000	0.040	0.075	0.113	0.041	0.042	0.000	0.000	0.051	0.139	—	—	—	—	—
1997–98	—	—	0.000	0.004	0.018	0.099	0.071	0.048	0.122	0.011	0.003	0.046	0.151	—	—	—	—	—	—
1998–99	—	0.000	0.003	0.025	0.055	0.107	0.021	0.114	0.105	0.024	0.024	0.037	—	—	—	—	—	—	—
1999–00	0.000	0.001	0.051	0.008	0.069	0.109	0.037	0.139	0.084	0.035	0.040	—	—	—	—	—	—	—	—

Table 9: Proportions at age and sex used in previous hoki stock assessments for the commercial catch on the Chatham Rise and the Sub-Antarctic.

Chatham Rise Males

	1995	1994	1993	1992	1991	1990	1989	1988	1987	1986	1985
1991–92	—	—	—	—	—	—	0.004	0.001	0.330	0.070	0.014
1992–93	—	—	—	—	—	0.016	0.005	0.004	0.282	0.030	—
1993–94	—	—	—	—	0.038	0.117	0.010	0.040	0.173	—	—
1994–95	—	—	—	0.057	0.180	0.075	0.051	0.084	—	—	—
1995–96	—	—	0.074	0.220	0.078	0.028	0.023	—	—	—	—
1996–97	—	0.136	0.119	0.153	0.023	0.015	—	—	—	—	—
1997–98	0.029	0.260	0.050	0.029	0.056	—	—	—	—	—	—

Chatham Rise Females

	1995	1994	1993	1992	1991	1990	1989	1988	1987	1986	1985
1991–92	—	—	—	—	—	—	0.005	0.001	0.357	0.129	0.076
1992–93	—	—	—	—	—	0.022	0.001	0.024	0.480	0.120	—
1993–94	—	—	—	—	0.047	0.171	0.024	0.480	0.120	—	—
1994–95	—	—	—	0.070	0.173	0.096	0.088	0.128	—	—	—
1995–96	—	—	0.074	0.210	0.076	0.023	0.060	—	—	—	—
1996–97	—	0.166	0.268	0.051	0.053	0.016	—	—	—	—	—
1997–98	0.034	0.305	0.083	0.049	0.098	—	—	—	—	—	—

Sub-Antarctic Males

	1995	1994	1993	1992	1991	1990	1989	1988	1987	1986	1985
1991–92	—	—	—	—	—	—	0.000	0.002	0.168	0.050	0.215
1992–93	—	—	—	—	—	0.000	0.016	0.165	0.170	0.074	—
1993–94	—	—	—	—	0.006	0.034	0.027	0.265	0.178	—	—
1994–95	—	—	—	0.017	0.065	0.088	0.023	0.240	—	—	—
1995–96	—	—	0.012	0.046	0.228	0.128	0.146	—	—	—	—
1996–97	—	0.005	0.144	0.182	0.103	0.061	—	—	—	—	—
1997–98	0.011	0.190	0.121	0.090	0.111	—	—	—	—	—	—

Sub-Antarctic Females

	1995	1994	1993	1992	1991	1990	1989	1988	1987	1986	1985
1991–92	—	—	—	—	—	—	0.001	0.001	0.102	0.059	0.404
1992–93	—	—	—	—	—	0.003	0.006	0.102	0.134	0.277	—
1993–94	—	—	—	—	0.007	0.083	0.016	0.162	0.210	—	—
1994–95	—	—	—	0.015	0.079	0.074	0.001	0.398	—	—	—
1995–96	—	—	0.012	0.058	0.132	0.100	0.138	—	—	—	—
1996–97	—	0.003	0.070	0.133	0.119	0.177	—	—	—	—	—
1997–98	0.017	0.161	0.094	0.082	0.125	—	—	—	—	—	—

Table 10: Parameter estimates from the unstratified combined model.

	Growth parameters				Year effects		Method bias	
	Eastern stock		Western stock					
	Male	Female	Male	Female				
K	0.44	0.45	0.32	0.29	1991	1.63	CHAT bias age 1	2.18
t_0	-0.15	-0.08	-0.21	-0.26	1998	3.07	CHAT c.v. age 1	1.27
L_∞	70.49	72.67	87.12	92.80	1999	-0.80	CHAT bias age 2	1.03
$C_{K,1997}$	1.17		1.12				CHAT c.v. age 2	1.00
$C_{t_0,1997}$	0.16		0.13				SUBA bias age 1	-0.35
$C_{L_\infty,1997}$	-0.15		-0.13				SUBA c.v. age 1	0.70
c.v.	0.0465						SUBA bias age 2	2.26
							SUBA c.v. age 2	1.34

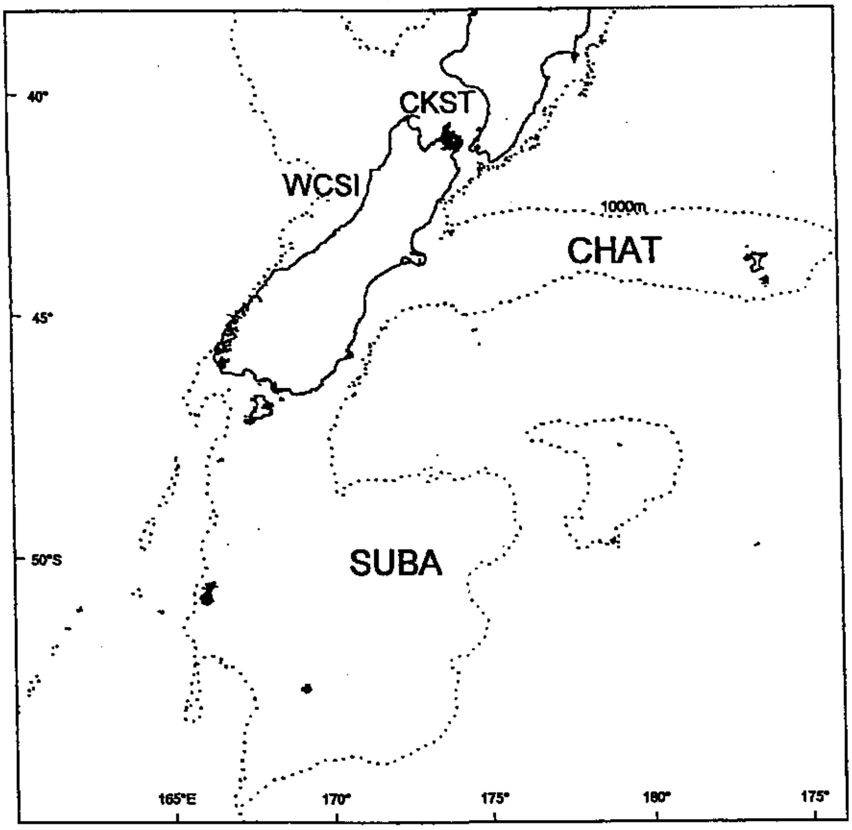


Figure 1: Location of the four main fishing areas for hoki. WCSI, west coast South Island; CKST, Cook Strait; CHAT, Chatham Rise; SUBA, Sub-Antarctic. The dotted line marks the 1000 m depth contour.

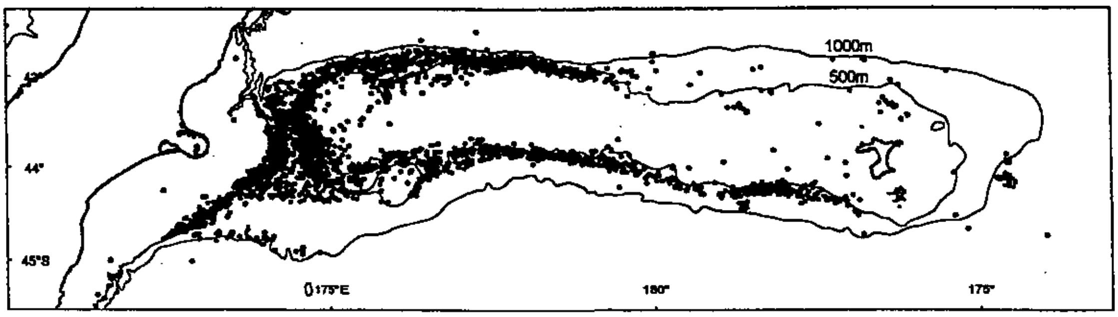


Figure 2: All observed commercial tows on the Chatham Rise for the fishing years 1991–92 to 1999–2000.

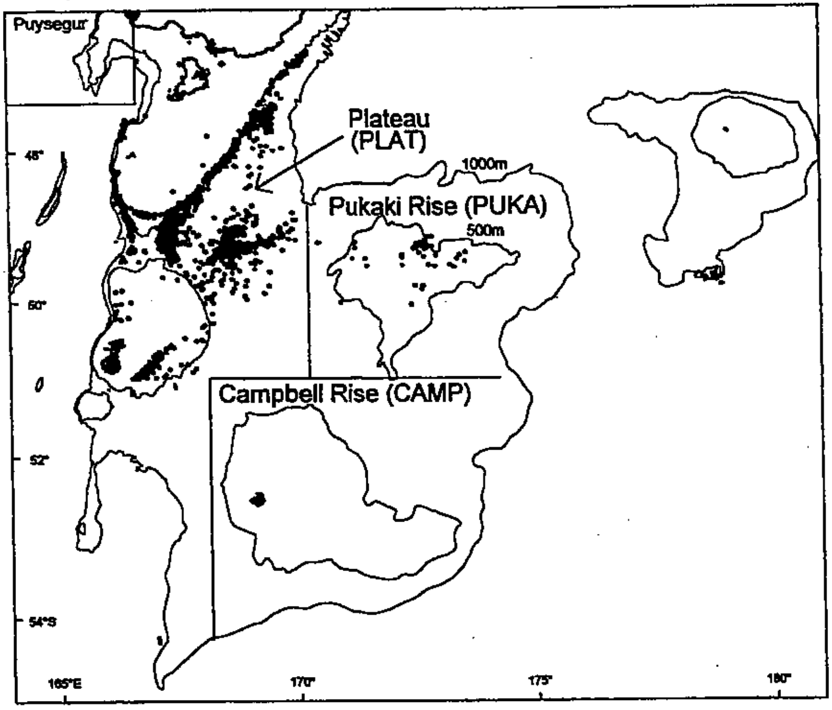


Figure 3: **All observed tows in the Sub-Antarctic region for the fishing years 1991–92 to 1999–2000. Commercially sensitive tows are not plotted.**

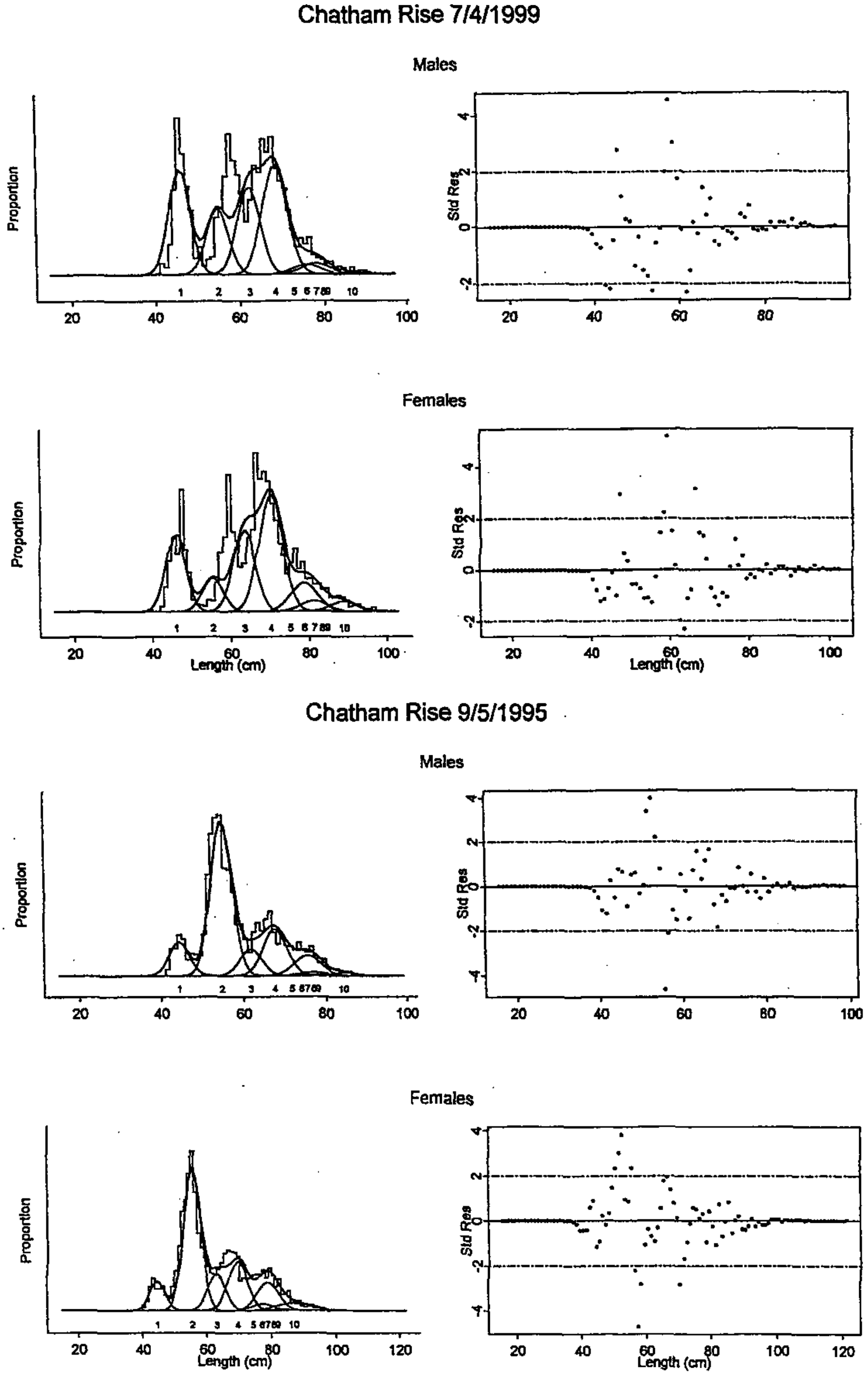


Figure 4: A comparison between fitted length frequencies for different times from the Chatham Rise. The 1997 cohort is age 2 in 1999. In the left side plots, the thin lines are the distributions of length at age, and the numbers above the x-axis indicate the mean length at that age. The plots on the right show the standardised residuals.

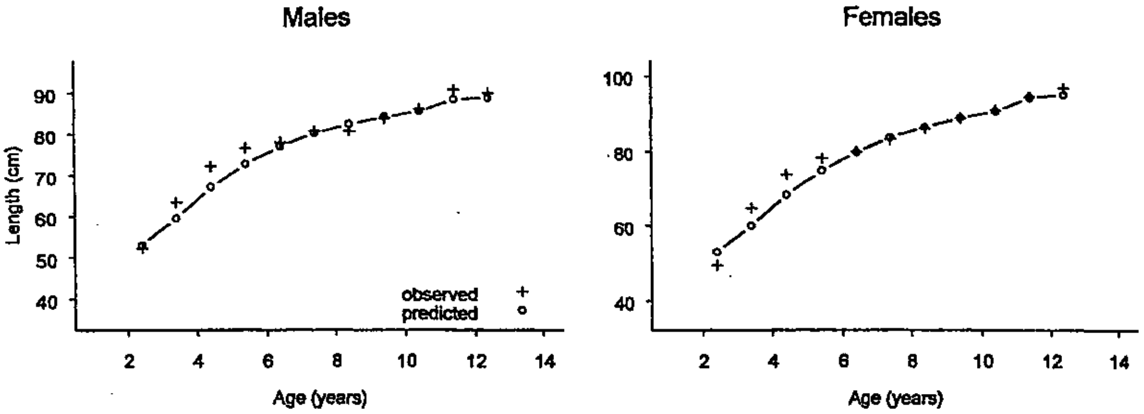


Figure 5: Observed mean lengths at age from the December 2000 Sub-Antarctic survey and the predicted mean lengths at age from the Sub-Antarctic model.

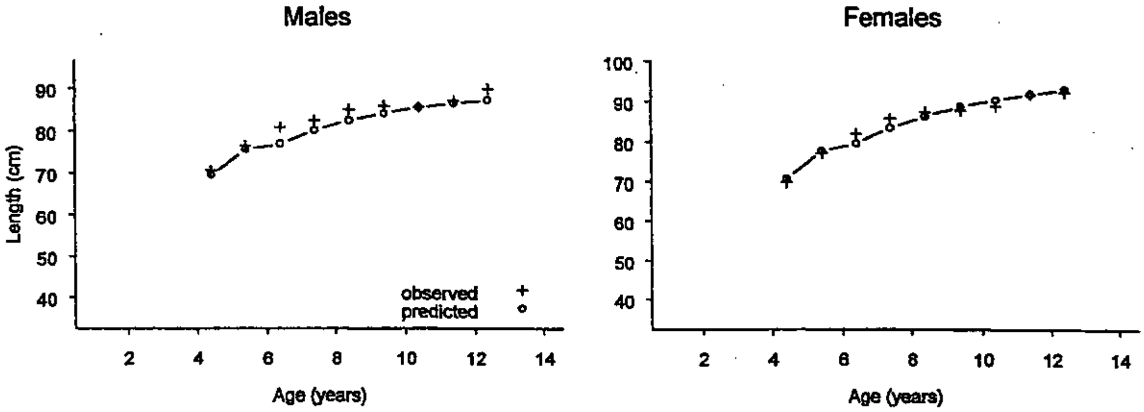


Figure 6: Mean lengths at age from the December 1993 trawl survey in the Sub-Antarctic. The 1988 and 1987 cohorts are ages 5 and 6 at this time. The predicted values are from the Sub-Antarctic model.

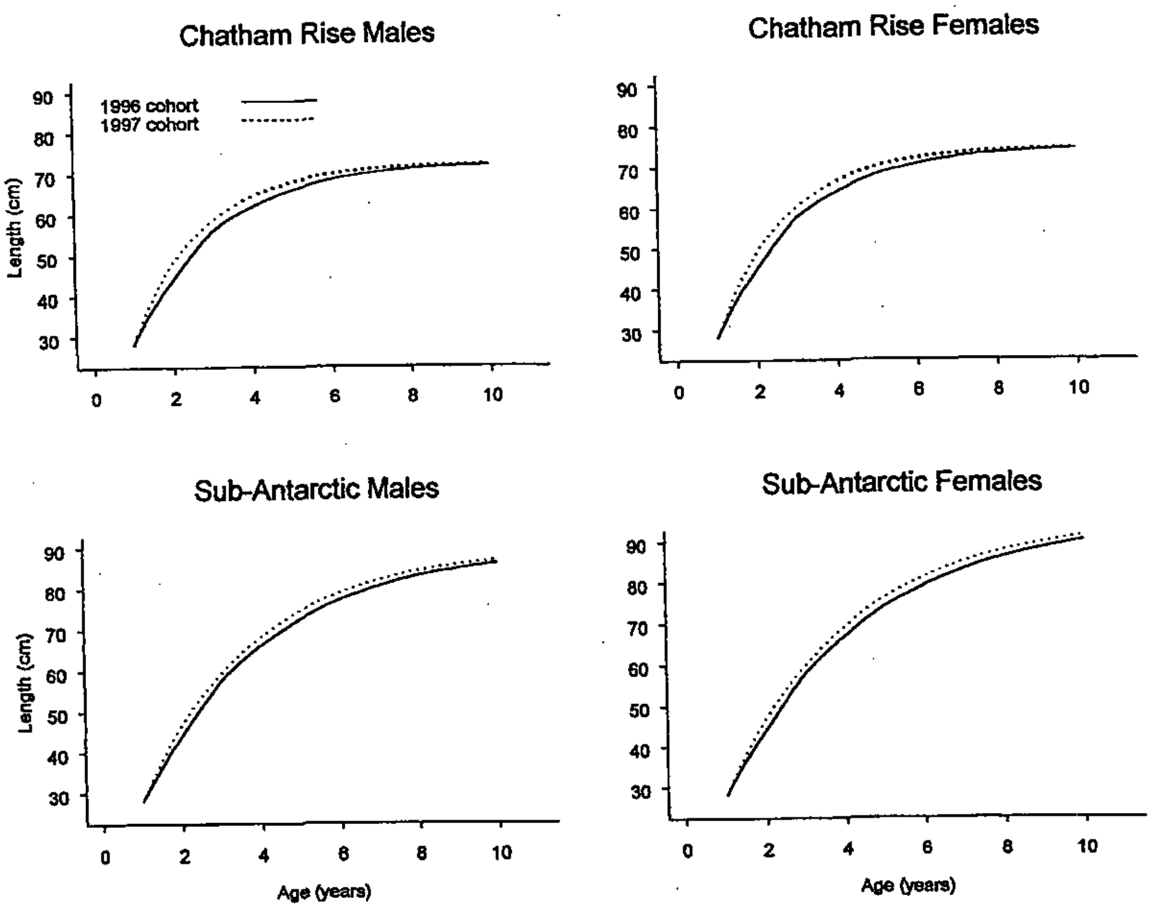


Figure 7: Estimated growth curves for the 1996 and 1997 cohorts from the combined model. The slight incongruity at age 3 for the 1996 cohort is due to the 1998 and 1999 year effects.

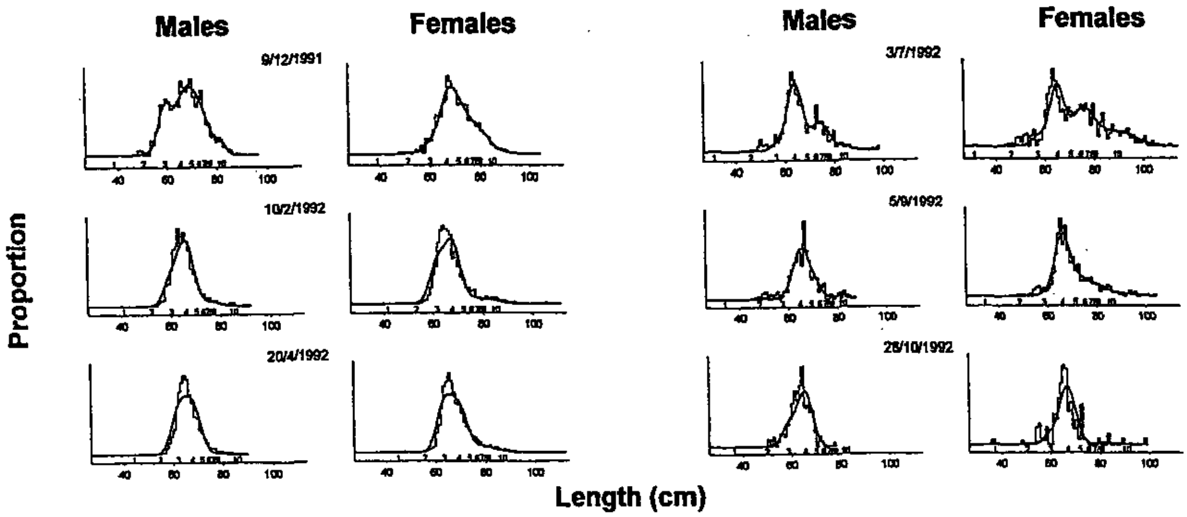


Figure 8: Fitted Chatham Rise length frequencies from the combined model.

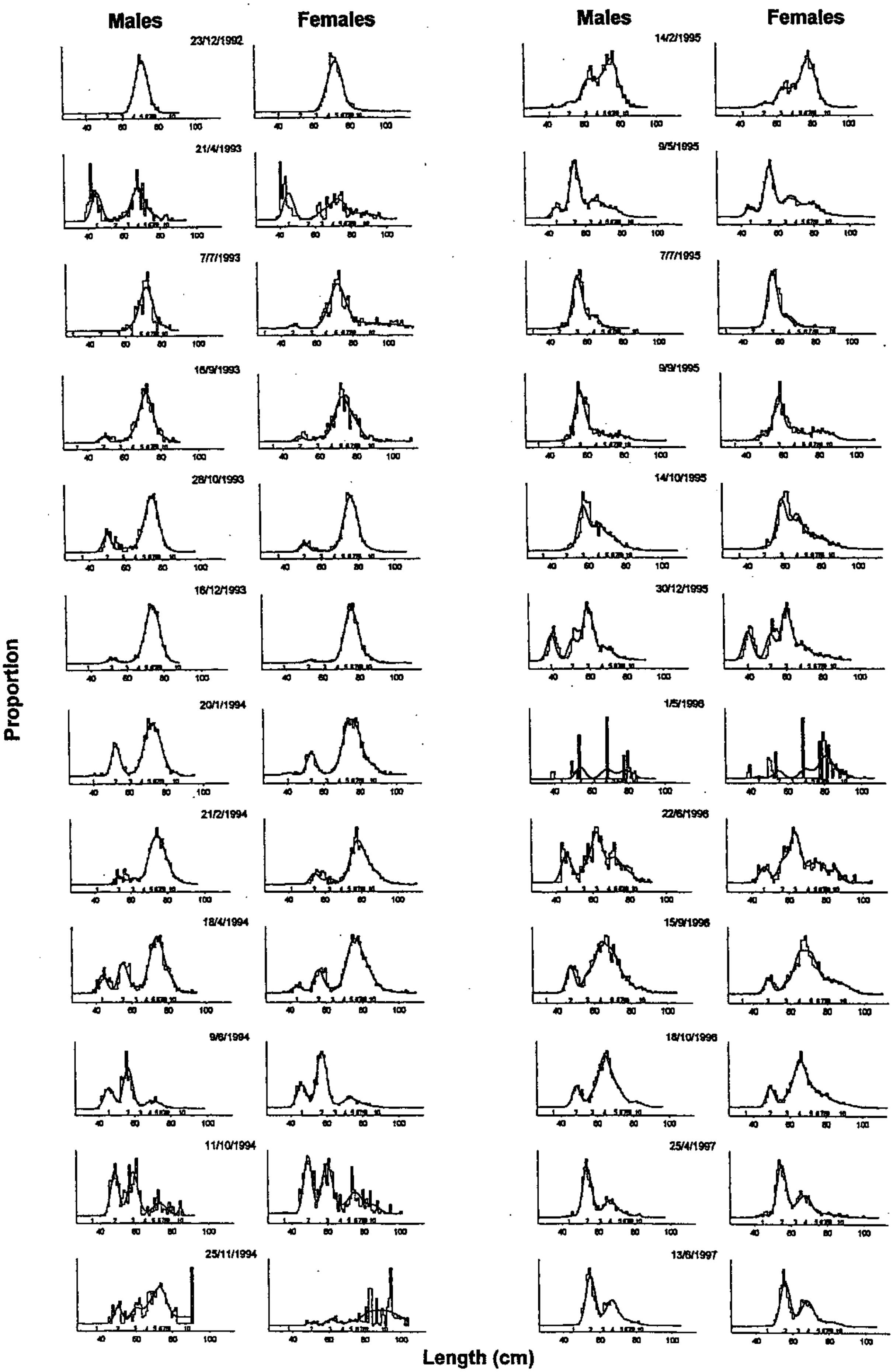


Figure 8: *continued*

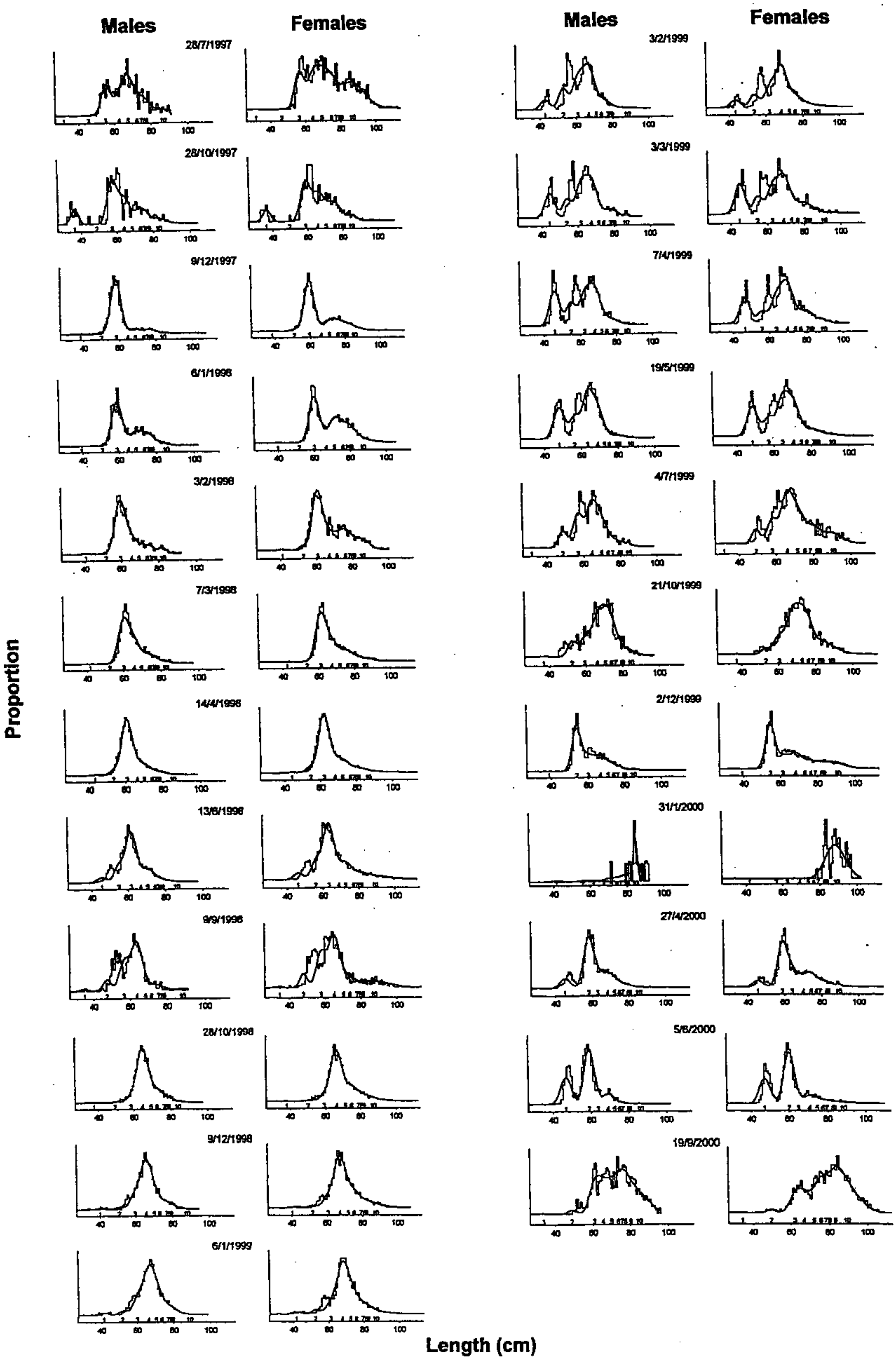


Figure 8: *continued*

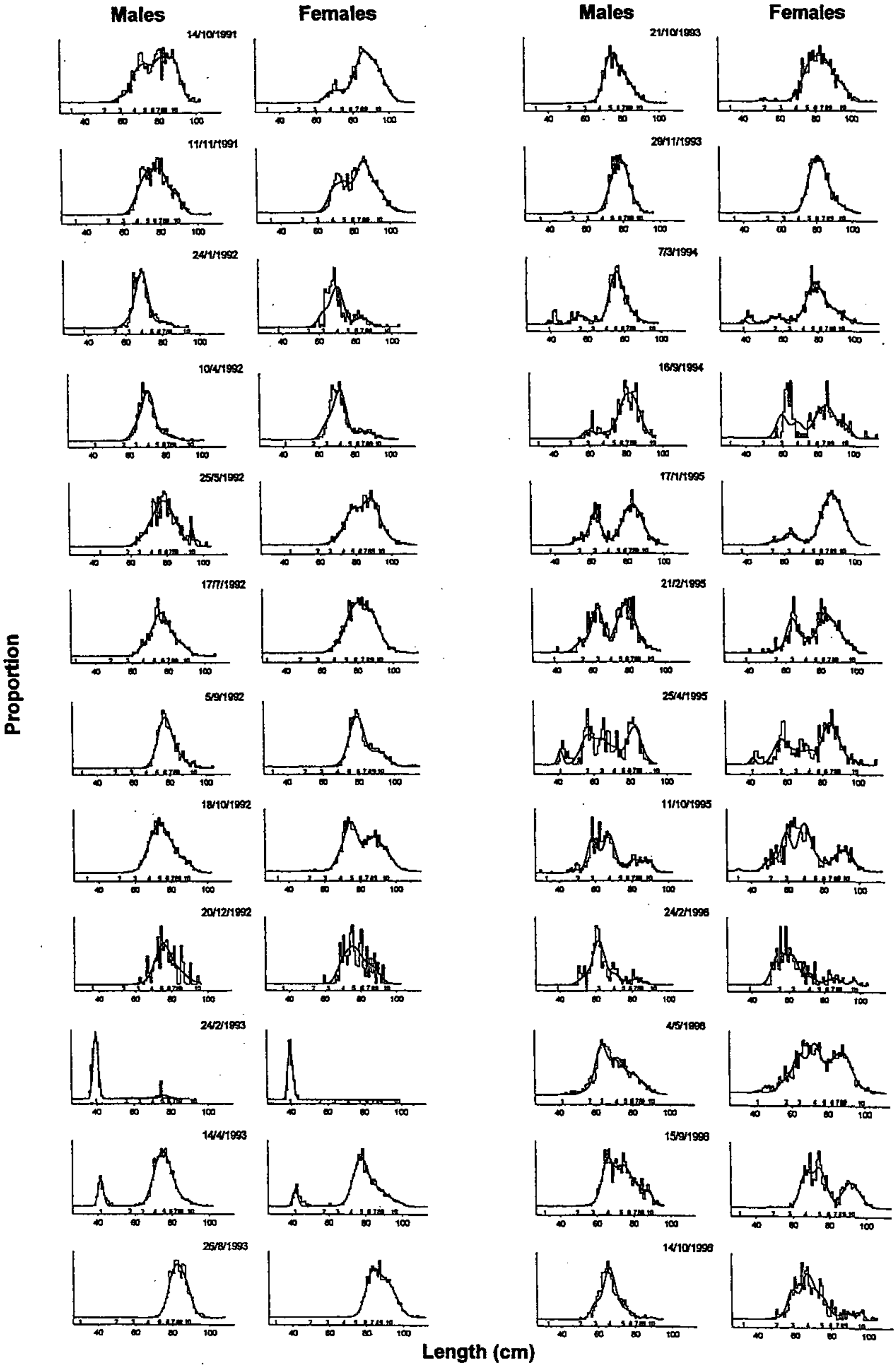


Figure 9: Fitted Sub-Antarctic length frequencies from the combined model.

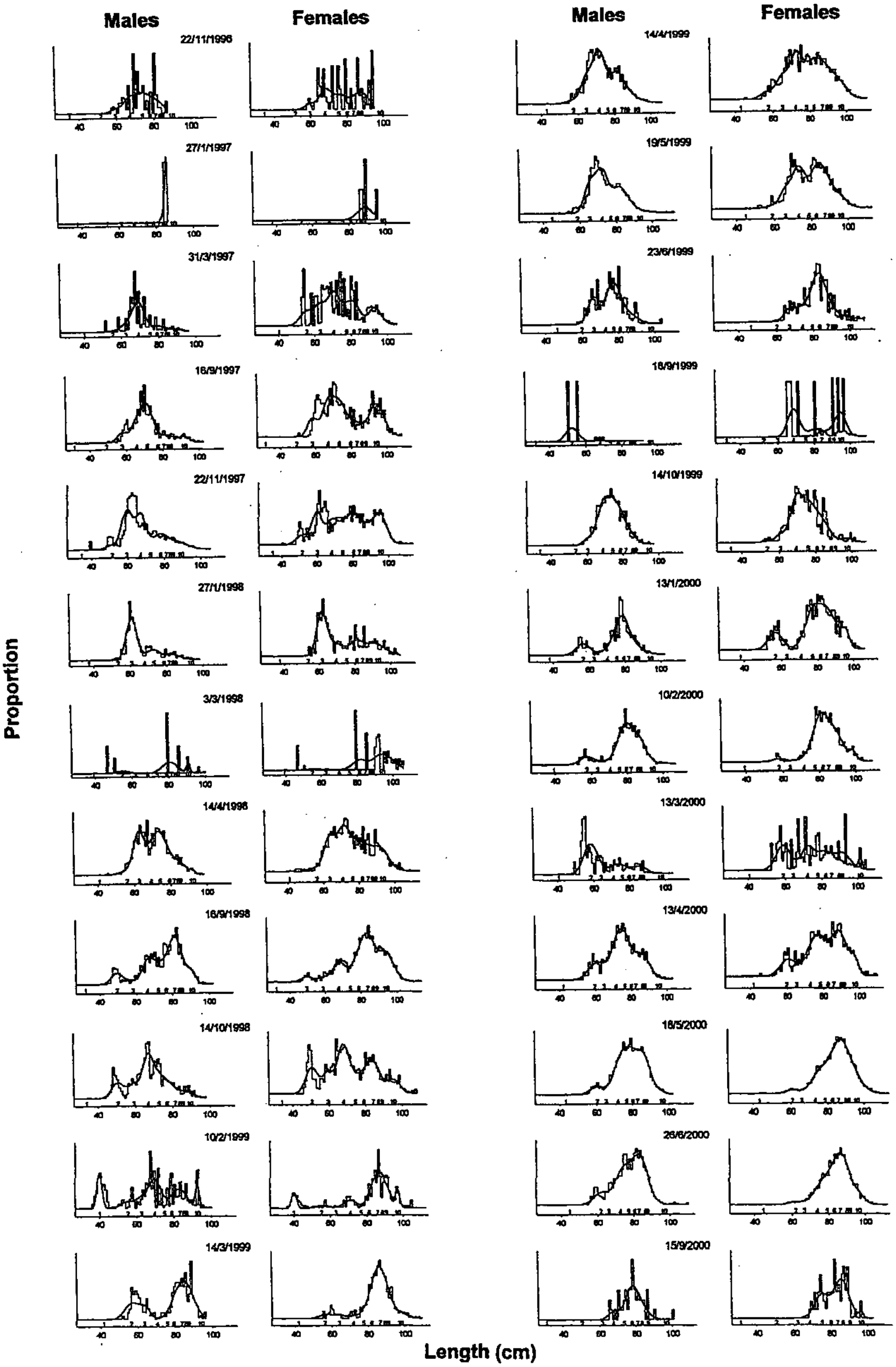


Figure 9: *continued*

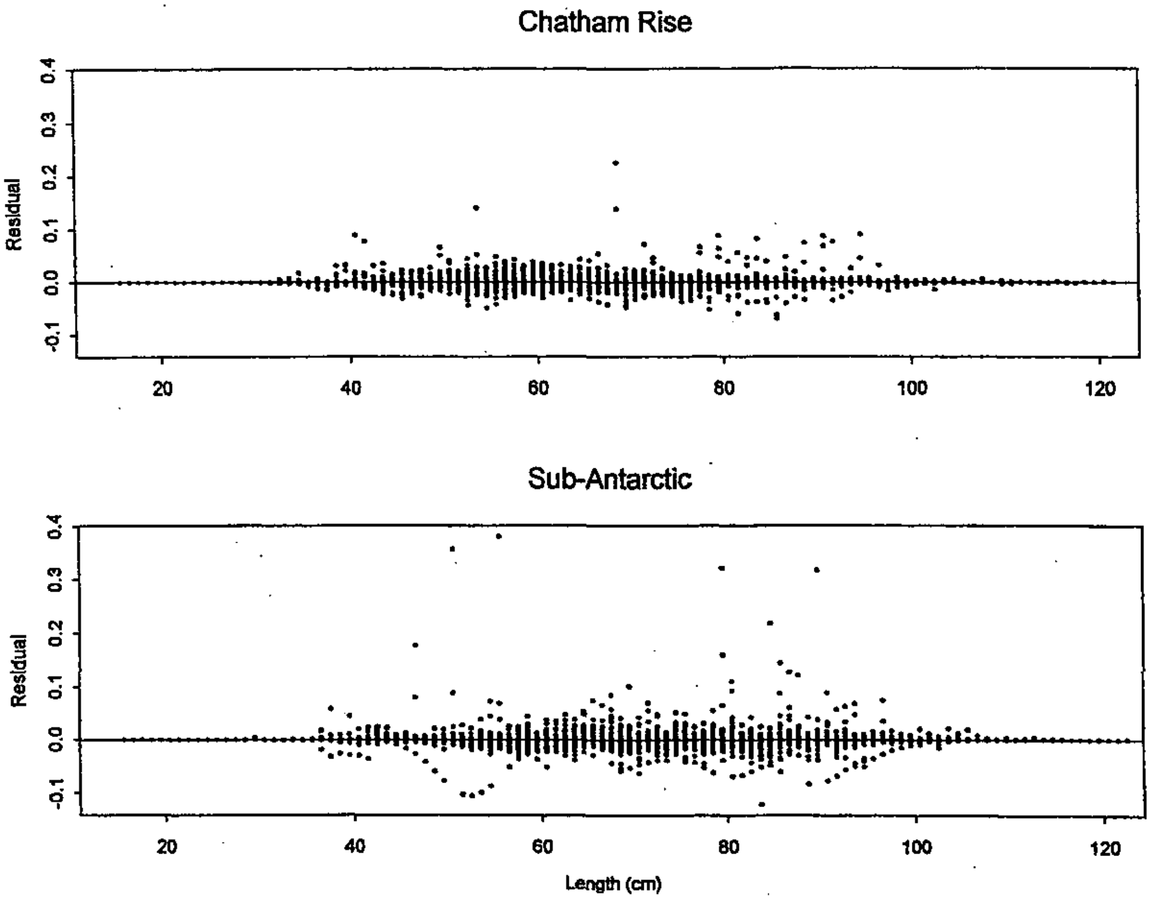


Figure 10: Residuals plotted against length for the predicted proportion at length in each area from the combined model. Positive residuals indicate under-prediction. The curve shaped trends are due to spiky observed length frequencies.

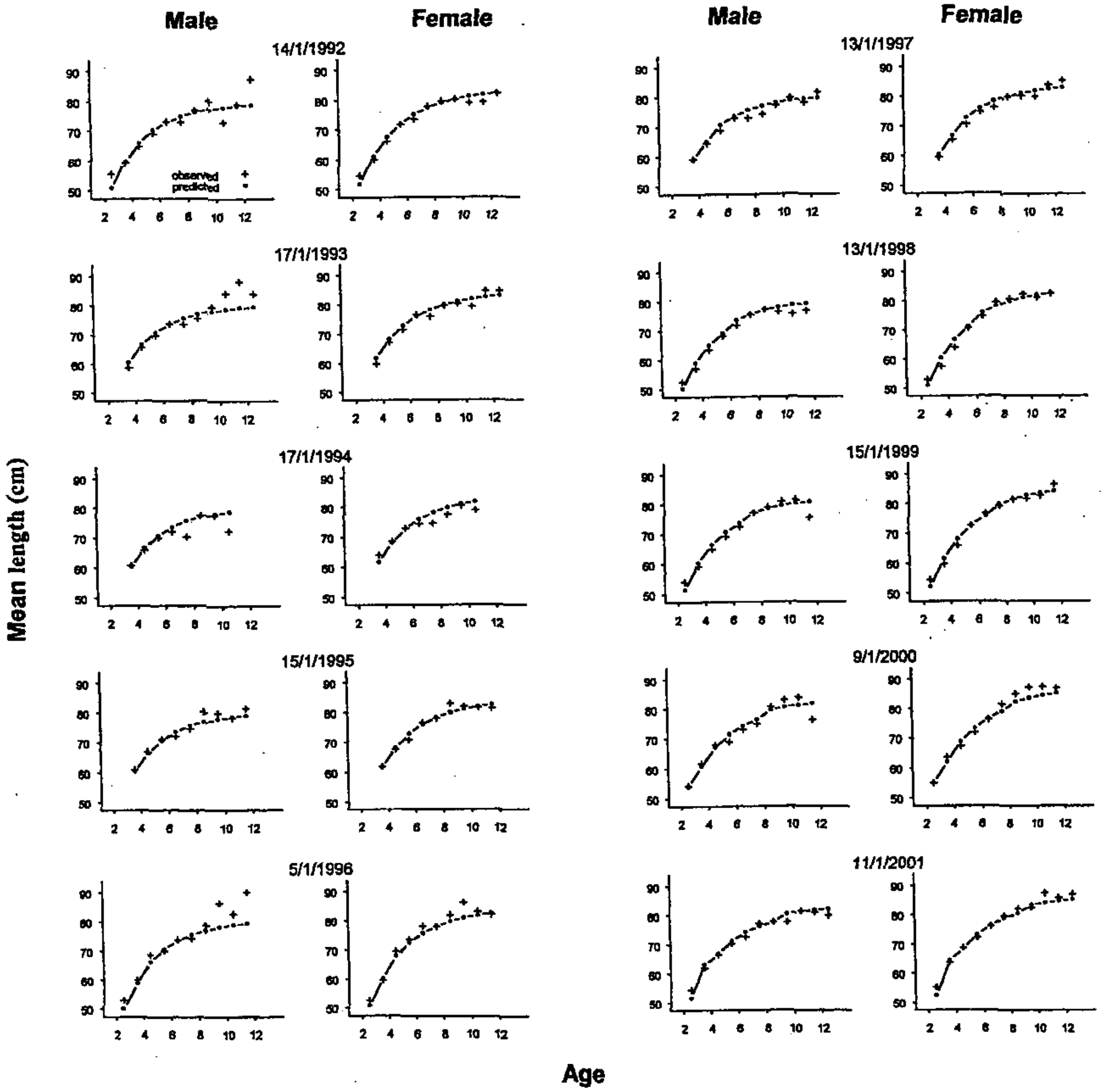


Figure 11: Observed and predicted mean length at age from the combined model for the Chatham Rise survey data. Crosses are observed values while circles are predicted mean lengths.

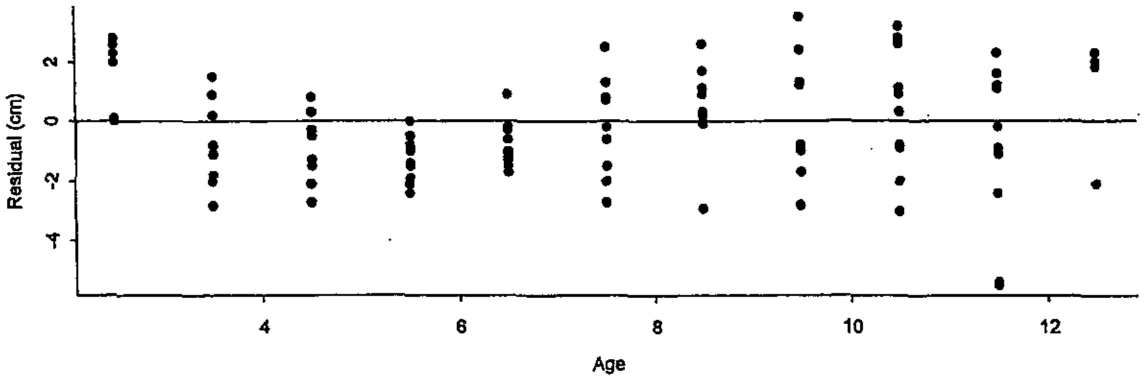


Figure 12: Residuals from the Chatham Rise mean length at age data plotted against age for the combined model. A positive residual represents an underprediction by the model. Residuals are in centimetres and are at the same age because all CHAT surveys occurred in January.

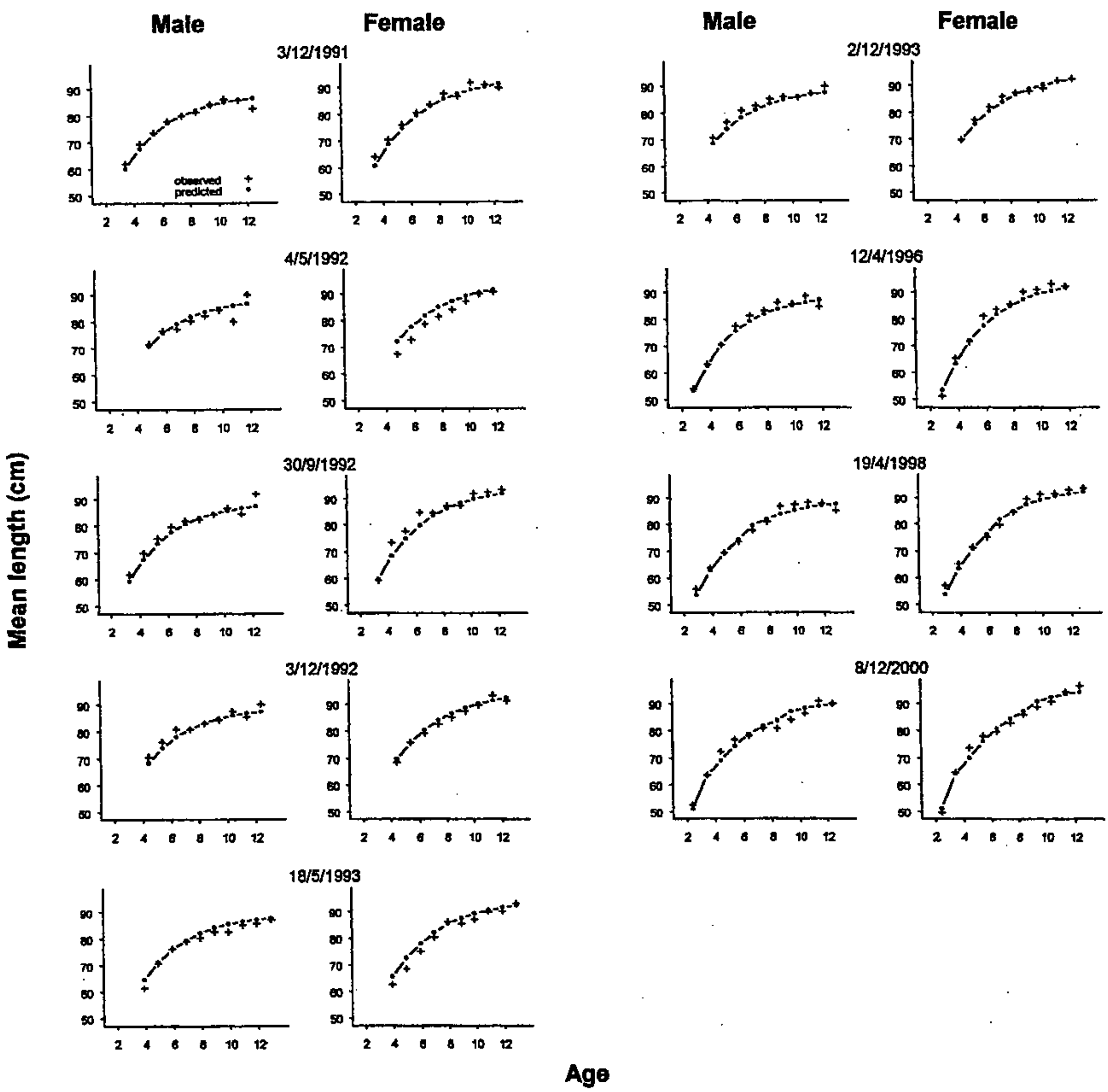


Figure 13: Observed and predicted mean length at age from the combined model for the Sub-Antarctic survey data. Crosses are observed values while circles are predicted mean lengths.

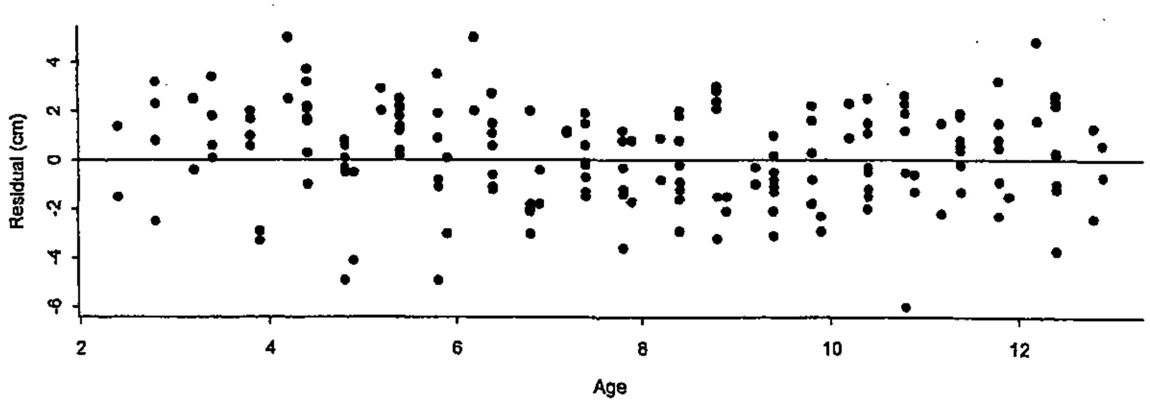


Figure 14: Residuals from the Sub-Antarctic mean length at age data plotted against age for the combined model. A positive residual represents under-prediction by the model. Residuals are in centimetres.

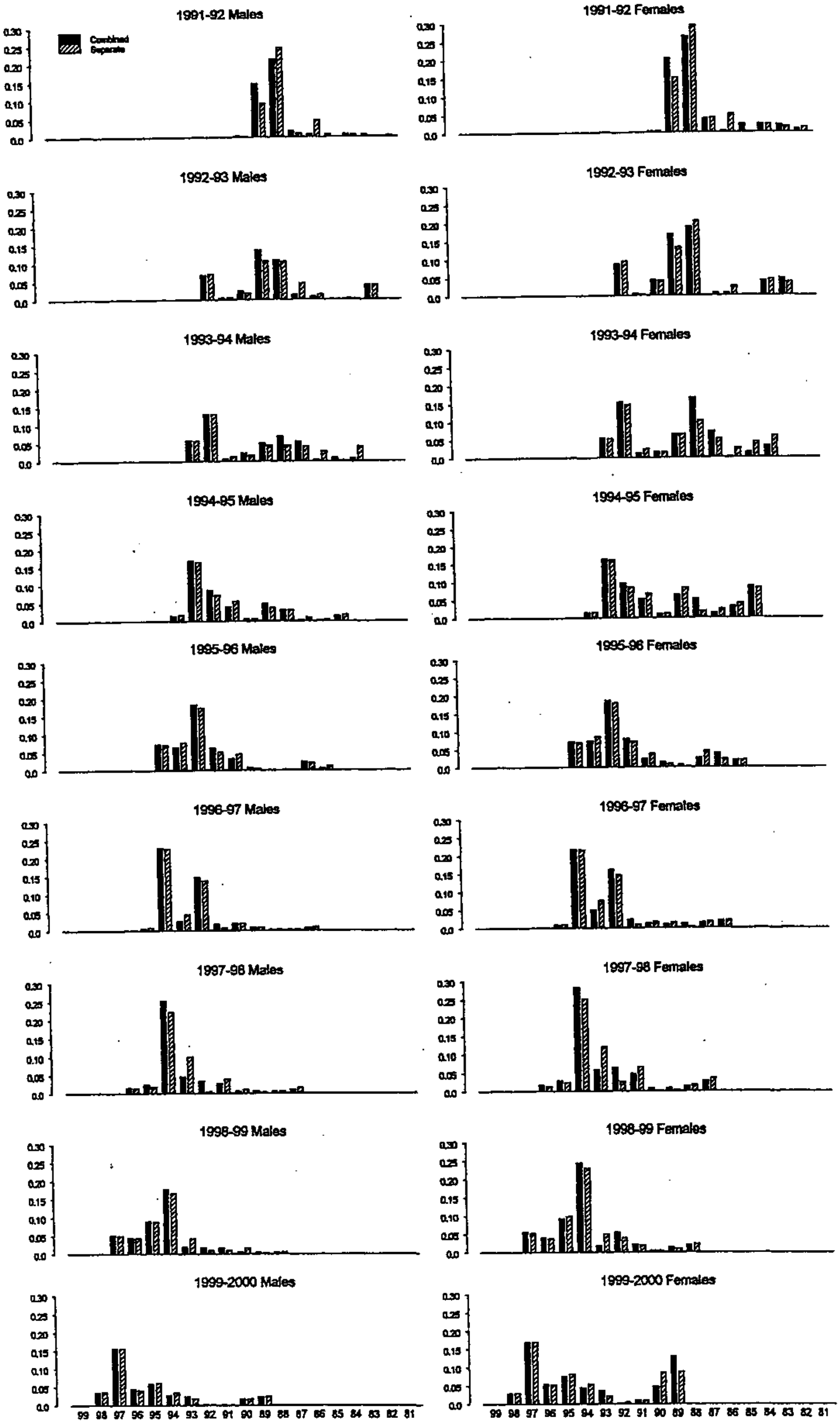


Figure 15: Estimated proportions of each cohort in the commercial catch from the Chatham Rise for each of the fishing years 1991–92 to 1999–2000 for the combined model and the Chatham Rise only model.

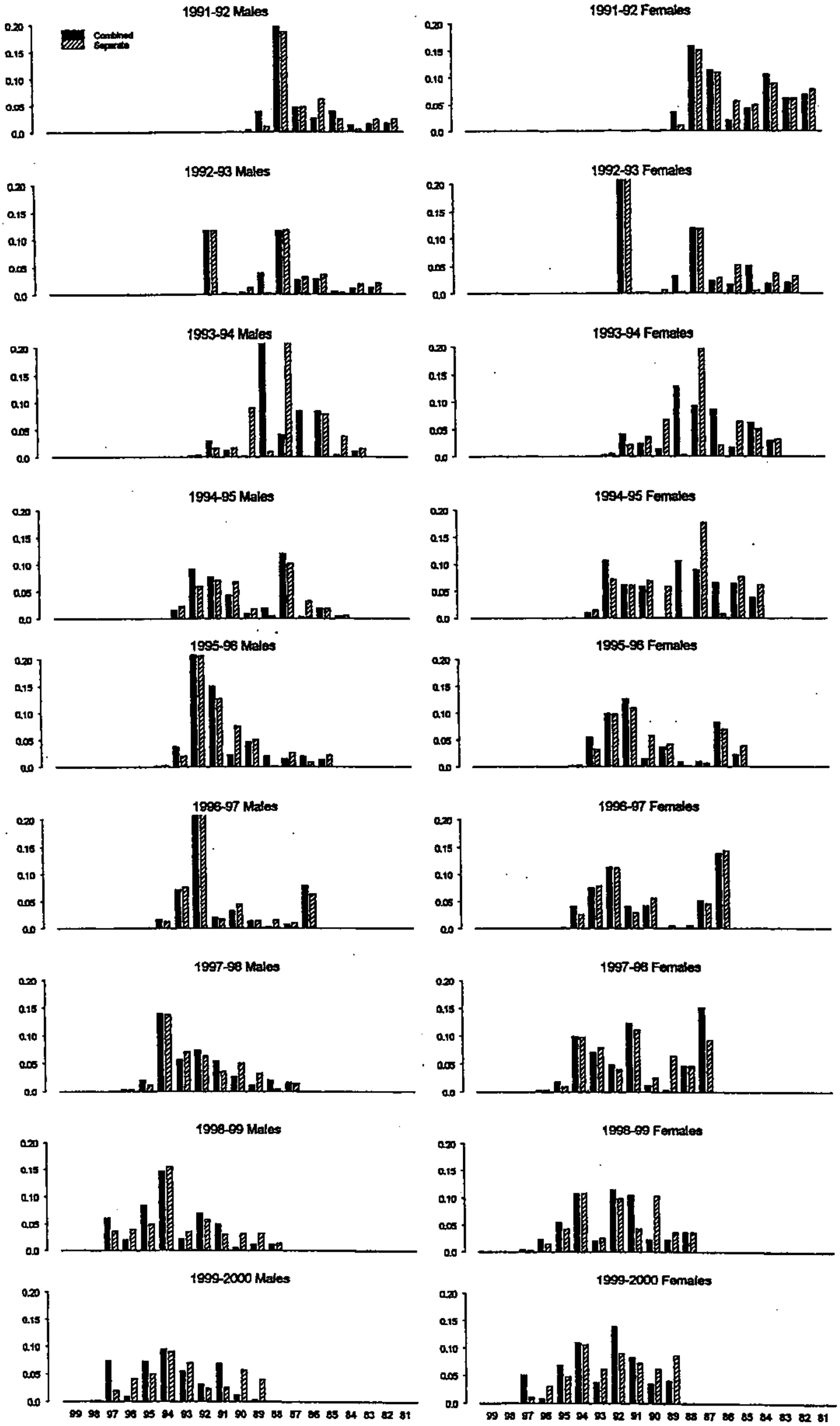


Figure 16: Estimated proportions of each cohort in the commercial catch from the Sub-Antarctic for each of the fishing years 1991–92 to 1999–2000 for the combined model and Sub-Antarctic only model.

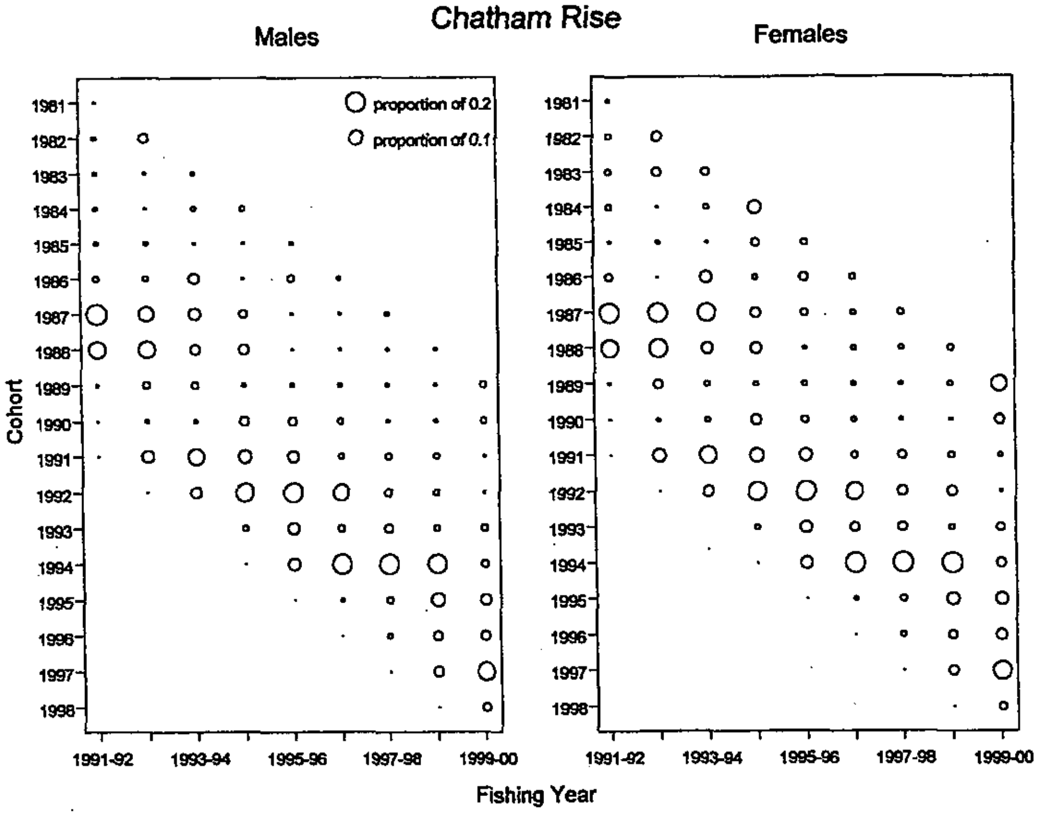


Figure 17: Proportions of each cohort and sex, represented by the area of the circles, on the Chatham Rise in the commercial catch for each fishing year as predicted by the combined model.

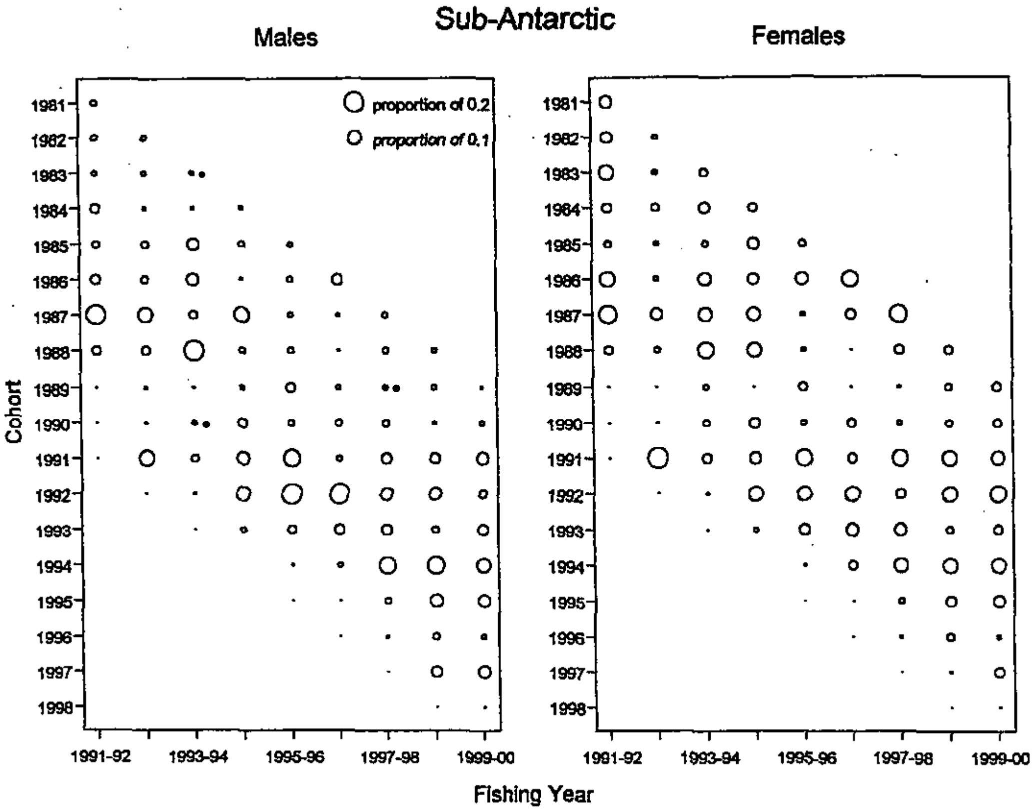


Figure 18: Proportions of each cohort and sex, represented by the area of the circles, in the commercial catch from the Sub-Antarctic for each fishing year as predicted by the combined model.

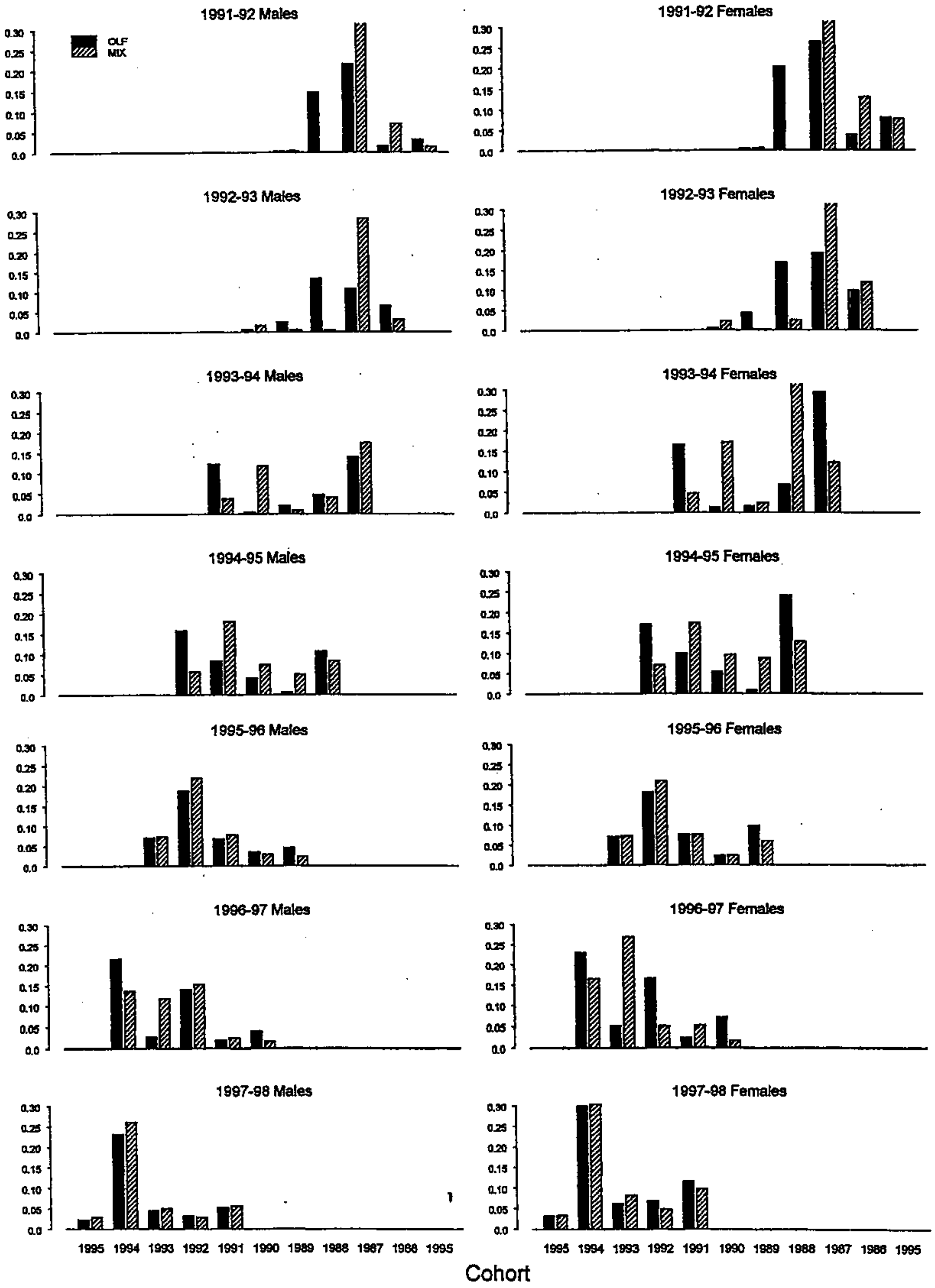


Figure 19: A comparison of the estimated proportions at age and sex from the combined model and the proportions at age and sex used in the previous hoki stock assessments for the Chatham Rise. The bars on the far right of each plot represent a plus group at age 6.

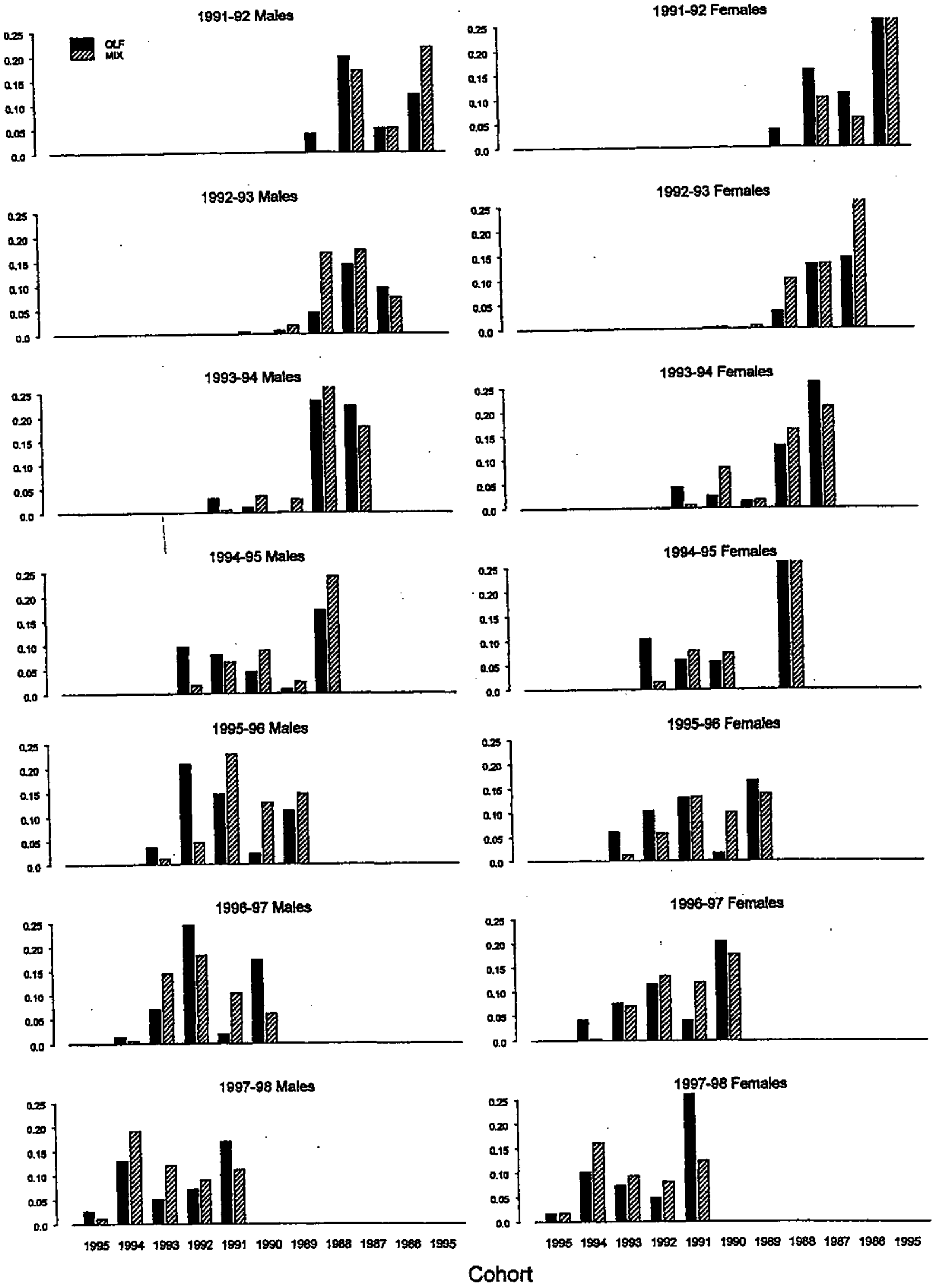
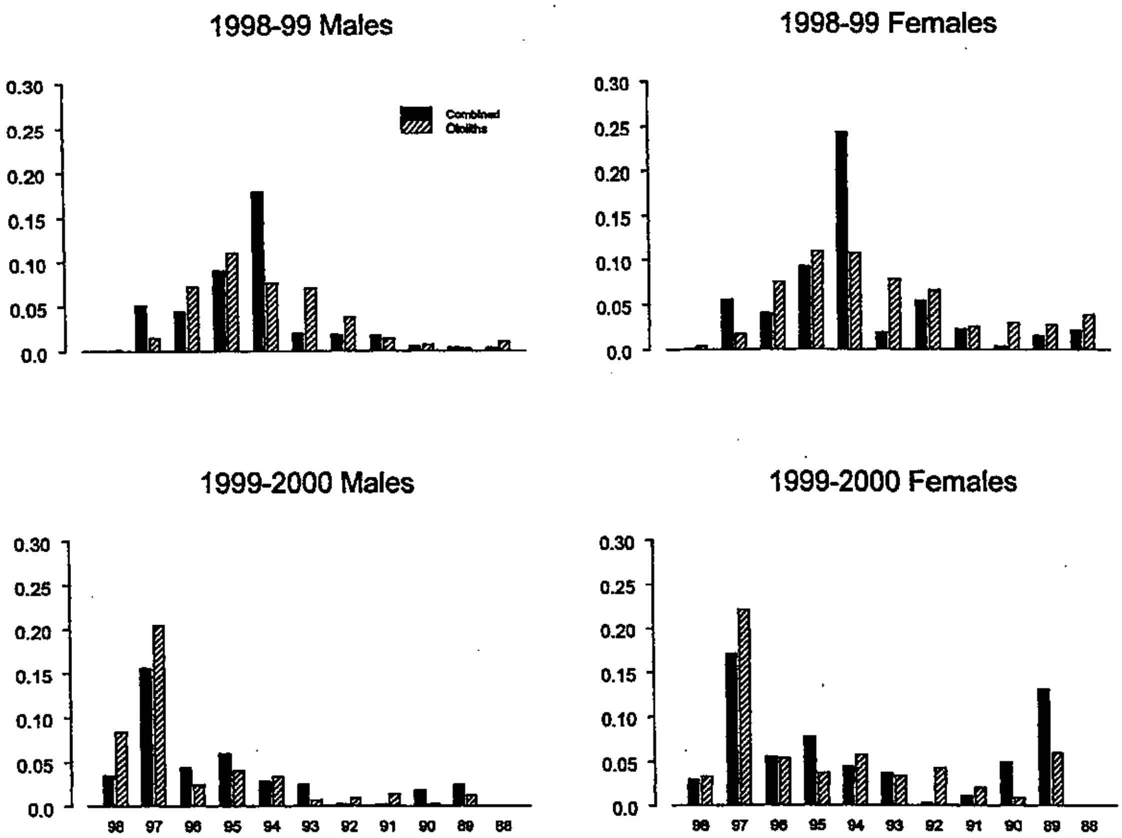


Figure 20: A comparison of the estimated proportions at age and sex from the weighted model and the proportions at age and sex used in the previous hoki stock assessments for the Sub-Antarctic. The bars on the far right of each plot represent a plus group at age 6.


1998-99 Males

1998-99 Females

1999-2000 Males

1999-2000 Females

Figure 21: A comparison between the proportions of each year class in the commercial catch estimated using OLF and using otolith ages (Francis 2002) for the 1998–99 and 1999–2000 fishing years.

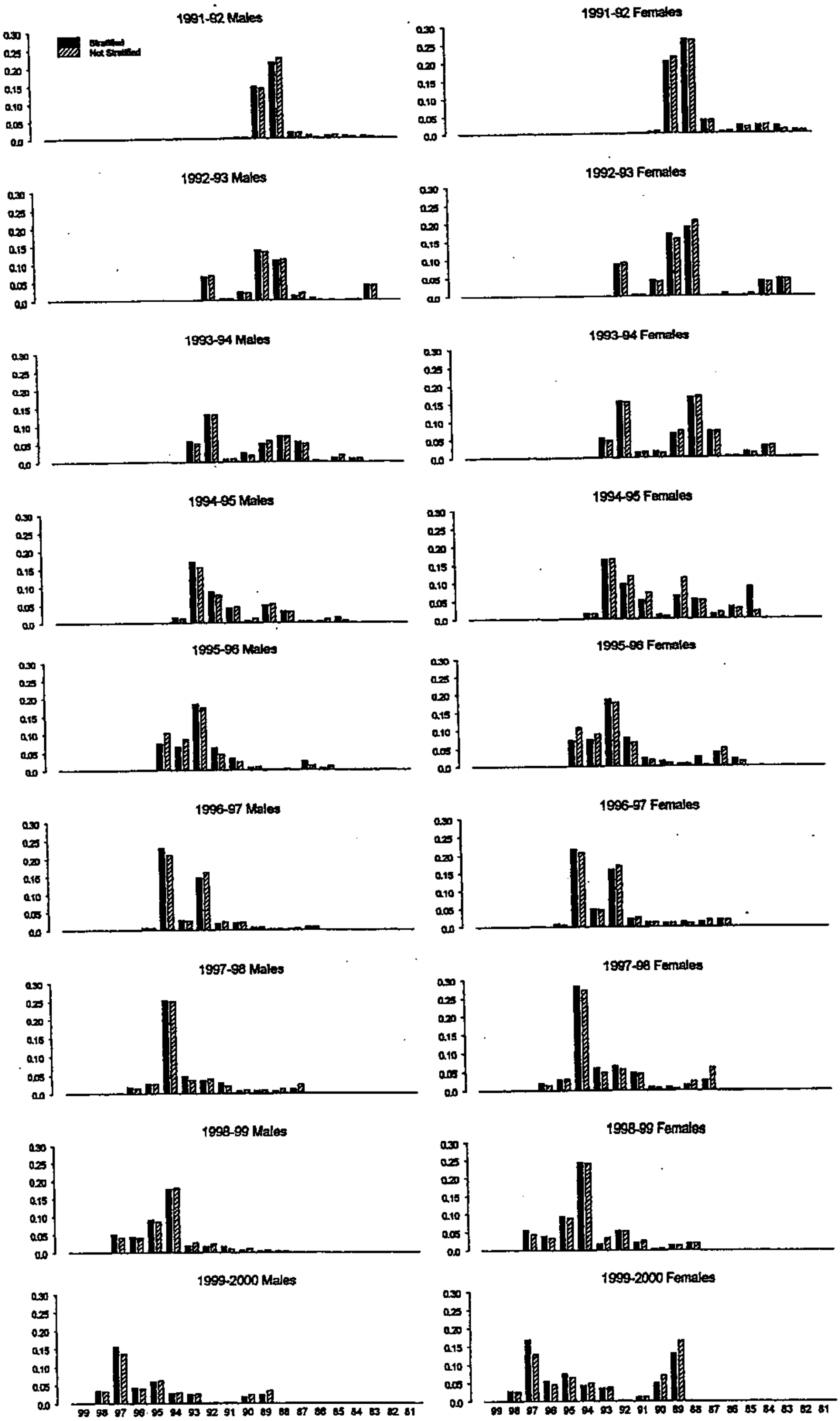


Figure 22: **Estimated proportions of each cohort in the commercial catch from the Chatham Rise for each of the fishing years 1991–92 to 1999–2000 for the stratified combined model and the combined model not stratified within each time period.**

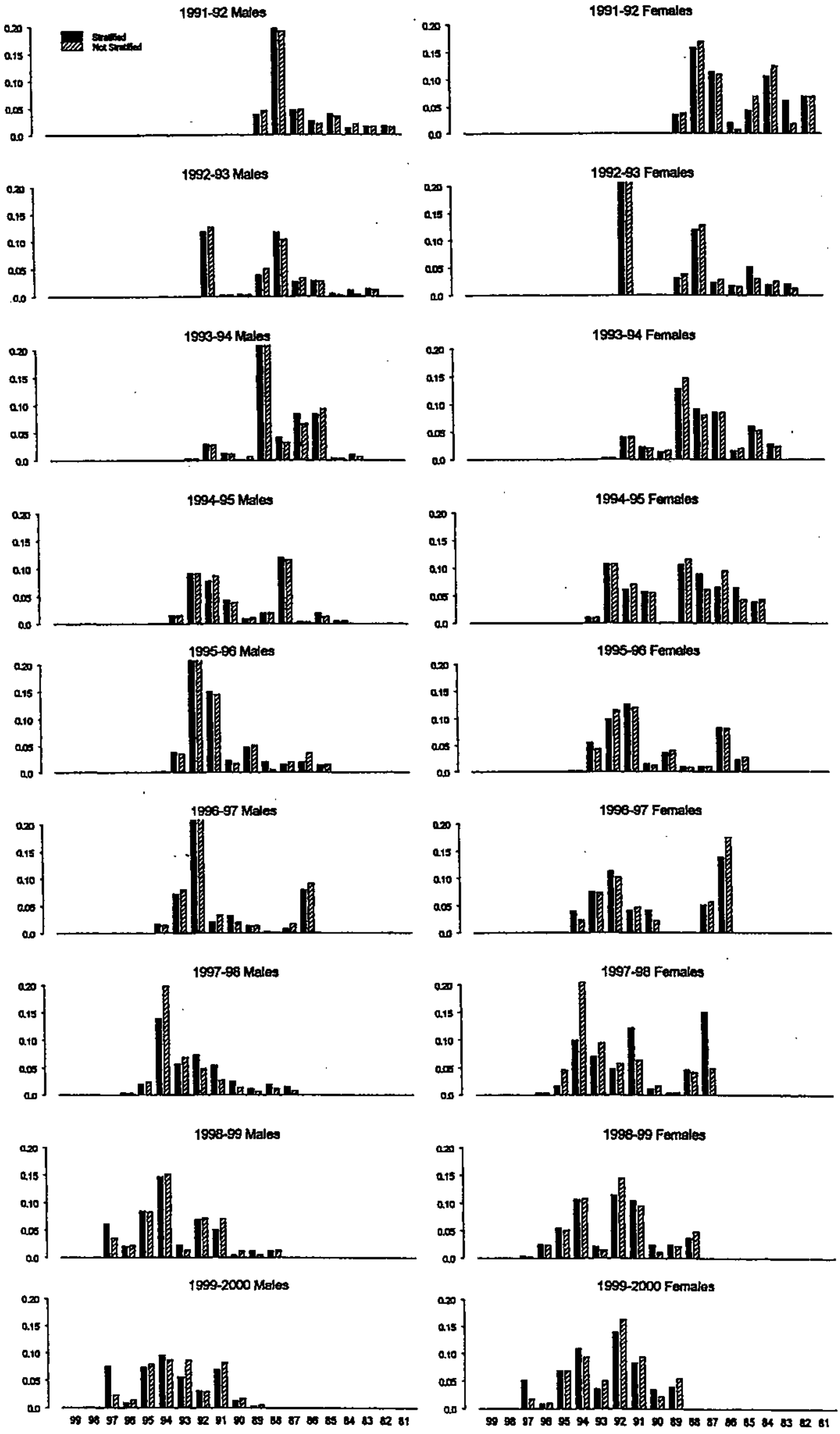
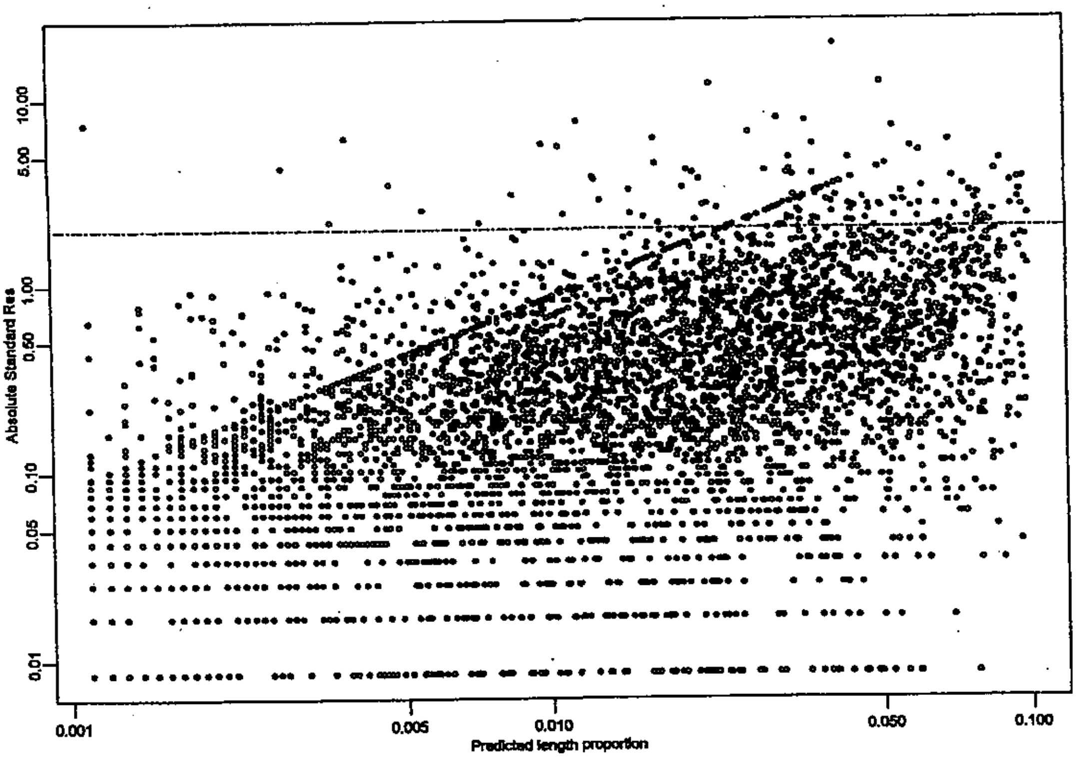
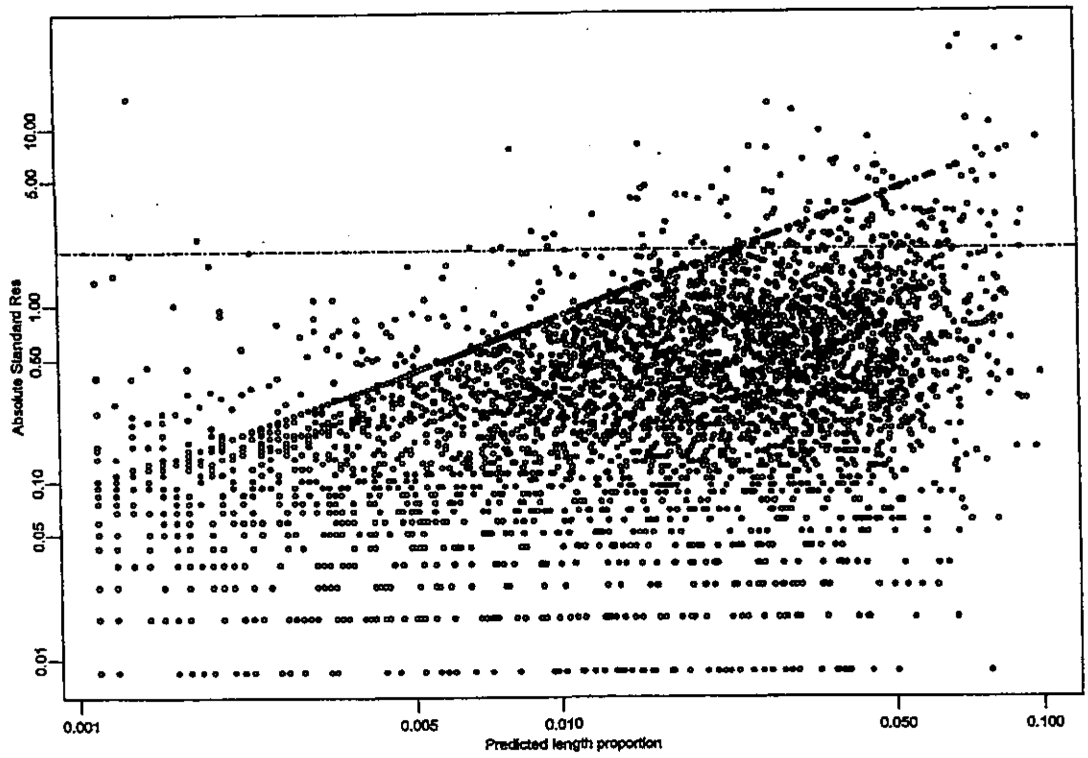


Figure 23: Estimated proportions of each cohort in the commercial catch from the Sub-Antarctic for each of the fishing years 1991–92 to 1999–2000 for the stratified combined model and the combined model not stratified within each time period.



Chatham Rise absolute standardised residuals



Sub-Antarctic absolute standardised residuals

Figure 24: Absolute standard residuals plotted against the predicted proportion of each length for the Chatham Rise and Sub-Antarctic, on logarithmic axes. The thick line is a loess line to show the trend in the residuals. The dashed line simply shows where the value 2 is on the y-axis.

APPENDIX 1: MEAN LENGTH AT AGE

The mean length at age for each sex was calculated as follows.

1. An age-length-key matrix was generated from the otolith age data. The row corresponding to each length was populated with the proportions-at-age for that length. Fish of lengths below the shortest length represented in the otolith data were assumed to all be 1-year-olds; lengths above the longest length in the otolith data were omitted from the age-length key. Rows corresponding to other lengths for which there were no age data were calculated as the average of the next highest and next lowest rows for which there were ages.

2. A matrix of numbers-at-length-and-age N was calculated from the age-length-key matrix A and the length frequency L: $N_{la} = L_l A_{la}$.

3. Mean-lengths-at-age were estimated as

$$\overline{L}_a = \frac{\sum_l l N_{la}}{\sum_l N_{la}}.$$

APPENDIX 2: MODEL DESCRIPTION AND ESTIMATION PROCEDURE

There are two main aspects to the estimation procedure, the underlying growth equations and the least squares fitting method. A generalised von Bertalanffy growth equation was used to model cohort specific growth. A weighted least squares fitting procedure with several optional penalty functions was used to determine the parameters which provided the best fit to the observed length frequencies and mean lengths at age.

Growth equation

The standard von Bertalanffy growth equation for mean length at age was generalised to include possible seasonal and "year" effects (e.g., a "good" or "bad" growing year for all cohorts). The growth curves were allowed to differ across stocks, sexes, and cohorts. For a particular sex and stock, the mean length at age was calculated as:

$$L_\infty(age)\left[1 - e^{K(t_0-age)} e^{a\sin(2\pi age+b)}\right]$$

where

age = the age measured from a specified spawning date for the cohort
K, t_0 = stock and sex specific growth parameters
a, b = seasonal growth parameters (independent of cohort, sex, or stock).

Chosen cohorts may have different growth parameters from the other cohorts by adding in cohort effects. Each parameter was changed in the following way,

$$K_j = KC_{K,j}$$
$$t_{0,j} = t_0 + C_{t_0,j}$$
$$L_{\infty,j} = L_\infty + C_{L_\infty,j}$$

where C is the estimated effect for the associated growth parameter of cohort j.

Year effects consisted of adding a constant onto the L_∞ parameter for the chosen year.

$$L_\infty(age) = L_\infty + \sum_y I_y(age) Y_y$$

where

L_∞ = cohort specific growth parameter
y = year index (the summation being over all years)
Y_y = year effect in year y
$I_y(age)$ = 0 when $y <$ birthyear of cohort
= 0 when $y >$ birthyear of cohort + age of cohort in whole years (N)
= $age - N$ when y = birthyear of cohort + N
= 1 otherwise.

The *age* of a cohort may be a decimal number, thus partial year effects will be applied.

Estimation method

A weighted least squares estimation procedure was used with optional penalty functions. Two types of data were fitted: length frequencies and estimates of mean length at age. For each block of data (of each type), several codes were specified to describe the data. In particular, there were "stock-mix" and "method" codes. The stock-mix code indicated whether the data were collected from an area where the eastern and western stocks were mixed (Chatham Rise), and the method code ranged from 1 to 6 depending on where and how the data were collected (Chatham Rise and Sub-Antarctic trawl surveys, WCSI, Chatham Rise, and Sub-Antarctic observer data, Cook Strait shed sampling). For length frequency data, extra length bins each with a zero proportion were added to every block down to a specified minimum length (i.e., zero observations on the left of the length frequency were included in the fitting procedure).

Length frequencies were fitted assuming normal distributions for length at age for cohorts of the same age. When a plus group was fitted (being a combination of cohorts at different ages) a lognormal distribution was assumed. A common c.v. was estimated for the normal distributions, while the lognormal distribution of the plus group had its own estimated c.v.. For some or all of the methods, a "bias adjustment" for the mean length of each age, up to a specified maximum age, could optionally be estimated (this was to allow for differences in length based selection across "methods"). This was an additive parameter in each case. If a "bias adjustment" was estimated then an associated "method-adjusted c.v." was also estimated (as a multiplier of the cohort specific c.v.).

As mentioned earlier, cohort effects could be introduced that would change the mean length at age for specific cohorts. Independently chosen cohorts could also have a c.v. effect where the average c.v. for that cohort was multiplied by an adjustment parameter. With this formulation it was possible to make all the cohorts share the same mean length at age distributions, but there was the ability to free up some or all cohorts to have their own distributions. The year effects could also change the mean length at age by shifting L_∞ for all cohorts in that year. Predicted values for mean lengths at age were obtained for each cohort at a specific time from the growth model. When the data were from the Chatham Rise, where the two stocks mix, an average length was predicted for each cohort assuming 50% eastern hoki and 50% western hoki.

In summary, the parameters to be estimated by the model are the von Bertalanffy growth parameters for each sex and stock, possible cohort effects on each of those parameters, possibly a year effect for every year in the model, a common c.v. for the normal distributions and an c.v. for the plus group's lognormal distribution, possible cohort specific c.v. effects, method bias and c.v. adjustments up to a specified age, and proportions at age for every length frequency.

The sum of squares to be minimised was:

$$\sum_T W_t \sum_i \sum_n (O_{tin} - P_{tin})^2 + \sum_k s_k P_k$$

where

t = index for observation type (1: length frequency, 2: mean length at age)
W_t = specified weight for observation type t
i = index for ith block
O_{tnj} = observed value: nth entry in ith block of observation type t
P_{tin} = predicted value: nth entry in ith block of observation type t
k = index for penalty functions
s_k = specified scalar for kth penalty function
P_k = penalty associated with kth penalty function

There were two observation types in the model to which the weights W_t apply. These are length frequencies and mean lengths at age and had respective weights of 10 000 and 1. These values equate a difference of 0.01 in the proportions (fitted in the lfs) with a difference of 1 cm in the mean lengths (i.e. 10 000*$(0.26–0.25)^2$ equals 1*$(56–55)^2$). The length frequencies were further down-weighted by 0.25 as explained in the text.

Predicted values for length frequencies (which were fitted as proportions) were produced using:

$$P_{1il} = \frac{\sum_{j} \alpha_{il} f(m_{il}, \mu'_{ij}, \sigma'_{ij})}{\sum_{l}\sum_{j} \alpha_{ij} f(m_{il}, \mu'_{ij}, \sigma'_{ij})}$$

where

j = index for cohorts
α_{ij} = proportion of *k*th cohort in the *i*th length frequency
f = pdf of normal or lognormal (plus group only) distribution
l = index for length bins
m_{il} = mid point of *l*th length bin for *i*th length frequency
μ'_{ij} = method adjusted mean length for *j*th cohort in *i*th length frequency
$\sigma'_{ij} = cv'_{ij}\,\mu'_{ij}$
cv'_{ij} = method adjusted c.v. for *j*th cohort in *i*th length frequency

The division by the sum over the length bins was done to ensure that the predicted values always summed to 1 (as the observed proportions do).

Optional penalty functions were available; a function could be omitted by setting the associated scalar to zero. The penalty functions are given below where *i* is used to index length frequency blocks, *s* to index stocks, *m* to index sexes, and *j* to index cohorts.

Cohort proportions sum to 1
A strong penalty was imposed to force the predicted cohort proportions to sum to 1 for each length frequency. This scalar was set to 100.

$$s_1\left[\log\sum_{j}\alpha_{ij}\right]^2$$

Restricted variation in L
When cohort specific growth parameters are allowed for cohorts which have only been observed when relatively young, it often happens that the best fit to the data is obtained for unrealistically low values of the von Bertalanffy parameter L_∞. To counter this effect a penalty on the variation about the mean value of L (the parameter used here) within each stock and sex was used with s_2 equal to 1.

$$s_2\sum_{smj}\left(L_{smj}-\bar{L}_{sm}\right)^2$$

Similar length at age 1 for both stocks

A potential penalty on between stock variation in the mean length at age 1 was available (A_{sjk} = mean length at age 1 for stock s, sex j, cohort k; 1 = eastern stock, 2 = western stock). A scalar of 10 was used.

$$s_3 \sum_{mj} \left(A_{1mj} - A_{2mj}\right)^2$$

Target length at age 1

The fitting procedure often needed "help" to determine the age associated with the smallest length modes (particularly when cohort specific growth parameters were used). This was done by specifying a "target length" at age 1 (T_1) and imposing a penalty for deviations from this target. A target length of 28 cm was used with a scalar of 5.

$$s_4 \sum_{smj} \left(A_{smj} - T_1\right)^2$$

Monotonic increasing length with age

For some parameter combinations, the sinusoidal parameterisation of seasonal growth can allow fish to shrink. This was strongly discouraged using a scalar of 100 and the equation

$$s_5 \sum_{smj} I_{smj} \left(z_{smj} - 2\pi a\right)^2$$

where

I_{smj} = 0 if $z_{smj} > 2\pi a$
= 1 otherwise.

Note, $z_{smj} > 2\pi a$ is a sufficient condition for mean length to be monotonic increasing with age (differentiate the growth function and solve for the derivative being greater than zero).

APPENDIX 3: CALCULATING PROPORTIONS OF A COHORT AND SEX FOR THE FISHING YEAR

To calculate the proportion of a cohort and sex over the entire fishing year requires the estimated proportions for each sex and time period within the year to be scaled up by the catches in that time period. First, the proportion of each cohort and sex in the time period must be known. Since the model output is the proportion at age for a given sex, the proportion at age and sex was calculated from the estimated proportions output by the model and the proportion of females in the observed length frequency from time period i (λ_i).

$$\hat{p}_{ias} = \hat{p}_{ia}\hat{\lambda}_i \qquad s = female$$
$$\hat{p}_{ias} = \hat{p}_{ia}(1 - \hat{\lambda}_i) \qquad s = male$$

where p refers to the proportion and the subscripts i, a, and s refer to the time period, age, and sex, respectively. The hats for the proportions are implicit as they were estimated in the model. The numbers at age for each sex in time period i was then calculated as

$$\hat{N}_{ias} = \frac{\hat{p}_{ias}W_i}{\sum\limits_{a,s}\hat{p}_{ias}w_s(l\,|\,a)},$$

using the weight of the total catch in period i (W_i) and the sex specific length-weight relationship for the mean length at age a, $w_s(l|a)$. The proportion of sex s and age a over all the time periods was simply calculated from the numbers at age and sex.

$$\hat{p}_{as} = \frac{\sum\limits_{i}\hat{N}_{ias}}{\sum\limits_{i}\sum\limits_{a,s}\hat{N}_{ias}}$$