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A length-based model for cockles on Snake Bank, Whangarei Harbour, 2002

Jeremy McKenzie, Martin Cryer, Paul Breen & Susan Kim

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Final Research Report

	Title:	A length-based model for cockles on Snake Bank, Whangarei Harbour, 2002
	Authors:	Jeremy McKenzie & Martin Cryer
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7. Executive Summary:

A stochastic, dynamic, length-based, observation-error, time-series model for cockles on Snake Bank, Whangarei Harbour, is described. The model is driven by reported landings, and fitted using Bayesian techniques to estimates of absolute biomass and length frequency distributions from roughly annual surveys on the bank since 1983, to periodic length frequency distributions from the commercial catch, and to length frequency distributions from roughly quarterly surveys on part of the bank between 1992 and 1995. Two alternative growth models are explored; one has normal error structure and allows negative growth whereas the other has log-normal error structure and does not allow negative growth. Both models seem over-prescribed and tests on MCMC chains suggested they did not converge on estimates for many important parameters. To explain rapid changes in biomass, both models favour implausibly high rates of natural mortality and growth. Neither model could resolve the apparent incompatibility of the biomass and length frequency data sets, and the results were very sensitive to the relative weights accorded each data set. Because of these problems, we conclude that the models do not provide a reasonable description of the observed data and we are not confident that the estimated parameter values are useful.

8. Objectives:

Only Objective 2 is pertinent to this report; the other objectives have been reported separately.

1. To estimate the size structure and absolute biomass of cockles on Snake Bank during the 2001/2002 fishing year. The target coefficient of variation (c.v.) of the estimate of absolute recruited biomass is 20%.
2. To complete the cockle stock assessment and estimate yields for cockles on Snake Bank for the 2002/2003 fishing year.
3. To estimate the size structure and absolute biomass of cockles in other areas within Whangarei Harbour during the 2001/2002 fishing year. The target coefficient of variation (c.v.) of the estimate of absolute recruited biomass is 20%.

9. Methods:

9.1 Background and previous stock assessments

Snake Bank is a sandbank of about 1.5 km^2 close to the mouth of Whangarei Harbour in northern New Zealand (Figure 1). A commercial fishery for cockles has operated on the bank since at least the early 1980s, and this is the only bank within the harbour where commercial fishing is permitted. There are other cockle beds in the harbour but none has a density of large cockles similar to that on Snake Bank (Cryer et al., 2003).

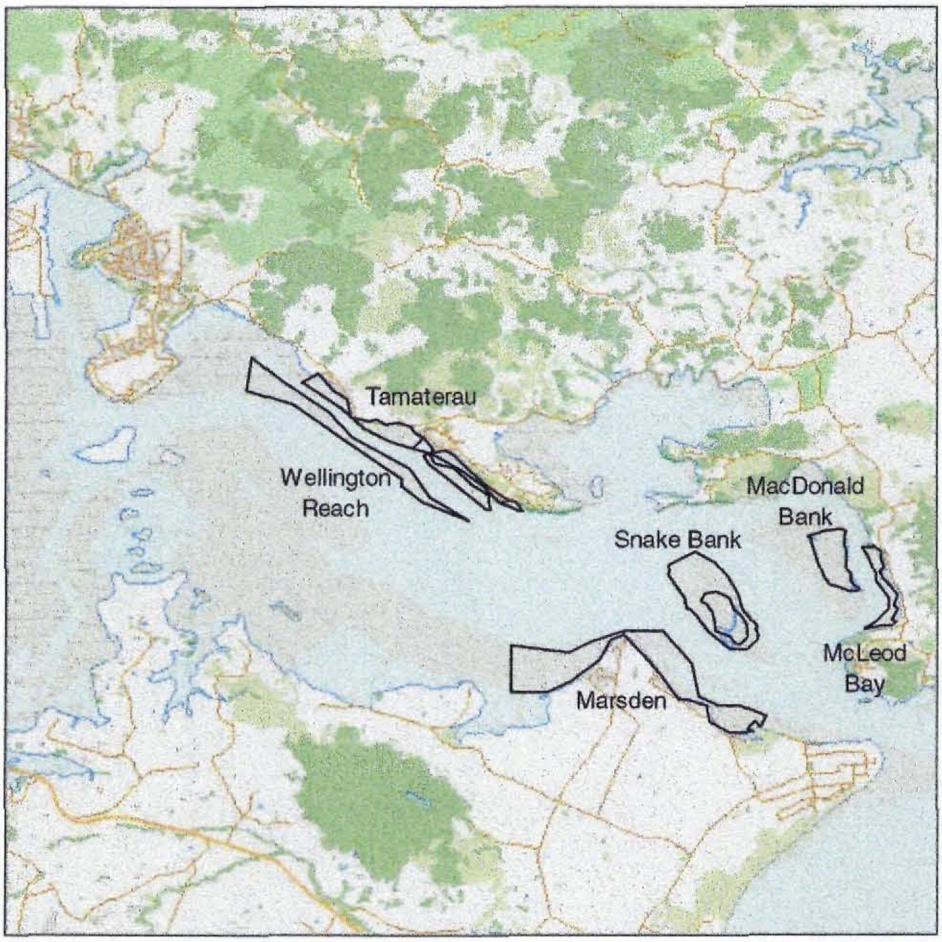

Figure 1: Beaches and banks within Whangarei Harbour, including Snake Bank. Unhatched grey indicates sandy, intertidal areas, and black polygons on these substrates indicate sampling strata from surveys on Snake Bank and elsewhere (after Cryer et al. 2002, 2003).

Surveys of the cockle population on Snake Bank have been conducted since 1982, some time after the first fishing, but well before a substantial fishery developed. Grid surveys were used until 1996 (Cryer, 1997), and various stratified random designs since (e.g., Morrison & Cryer 1999, Morrison 2000, Morrison & Parkinson 2001, Cryer & Parkinson 2001, Cryer et al. 2002). Whatever the design, the overall mean biomass of cockles on the bank (for a given size range) was estimated using the weighted average of the stratum estimates of mean biomass, weights being proportional to the relative area of each stratum:

$$\bar{x}_y = \sum_{i=1}^{n} W_i \bar{x}_i \tag{1}$$

where $\bar{x}_y$ is the overall mean biomass, W_i is the relative area and $\bar{x}_i$ the mean biomass in stratum i. These are considered estimates of absolute biomass (or abundance if estimated using numbers). Variance was estimated using:

$$s_y^2 = \sum_{i=1}^{n} W_i^2 s_i^2 / n_i \tag{2}$$

where s_y^2 is the variance of the estimated mean biomass, s_i^2 is the sampling variance in stratum i, and n_i is the number of samples taken in stratum i (Snedecor & Cochran 1989).

Station length frequency distributions were estimated by scaling the recorded length frequency distributions by the inverse of the sampled fraction at each station and to a square metre of sediment. Stratum length frequency distributions were estimated as the average station length frequency distribution for that stratum scaled by the stratum area (in square metres). The population length frequency was estimated by adding the stratum length frequency distributions.

Based on these surveys, yield for Snake Bank cockles was estimated by Cryer et al. (2002) as Maximum Constant Yield (MCY, method 2. equation 3) and Current Annual Yield (CAY, using the full version of the Baranov Catch Equation, equation 4, see also Annala *et al.* 2002).

$$MCY = 0.5.F_{0.1}.B_{av} \tag{3}$$

$$CAY = \frac{F_{ref}}{F_{ref} + M}\left(1 - e^{-\left(F_{ref}+M\right)}\right)B_{beg} \tag{4}$$

where $F_{0.1}$ and F_{ref} are reference rates of fishing mortality, M is natural mortality, B_{av} is the average recruited biomass between 1991 and 2001, and B_{beg} is the start of season recruited biomass. Estimates of $M = 0.30$, $F_{0.1} = 0.41$ (from Cryer 1997) were used. An estimate of $F_{max} = 0.62$ is available (Cryer 1997) but was not used.

The above describes a fairly crude stock assessment, and the Ministry's Shellfish Fishery Assessment Working Group decided that length-based modelling would be a better use of the data and should provide better estimates of yield (and projections of future stock status under different management regimes).

9.2 Input data for the length-based model

The following data were available for the length-based model: catch records since 1982 when the population was assumed to be close to virgin (Table 1); 13 estimates of absolute abundance (with variances) for cockles on Snake Bank between 1982 and 2002 (Table 2, Figure 2); 11 estimates of population length frequency distribution from surveys other than that in 1982 (Figure 3); 9 estimates of length frequency distribution from one part of the bank between August 1992 and March 1995 (Figure 4); and 4 estimates of the length frequency distribution of cockles taken by commercial fishers in 1992, 1996, and 2001 (early and late in the year, Figure 5).

Table 1: Reported commercial landings of cockles from Snake Bank since 1982 (after Annala et al., 2002) (values for 1982–86 and 2002 were assumed). No recreational or customary catch is included as these are thought to be very small compared with commercial landings (e.g., Annala et al., 2002, p. 112).

Year	Landings (t)	Year	Landings (t)
1982	162	1993	316
1983	162	1994	566
1984	162	1995	501
1985	162	1996	495
1986	162	1997	457
1987	114	1998	439
1988	128	1999	473
1989	255	2000	473
1990	426	2001	423
1991	396	2002	423
1992	537		

Table 2: Estimates of biomass (t) of cockles on Snake Bank from Cryer et al. (2002). Estimates marked with an asterisk (*) were made by analysis of length frequency distribution, others by directly weighing samples sorted into three size classes.

Year	n	Total		< 30 mm		> 30 mm		> 35 mm	
		Biomass	c.v.	Biomass	c.v.	Biomass	c.v.	Biomass	c.v.
1982	199	2 556	–	* 216	–	* 2 340	–	1 825	~0.10
1983	187	2 509	–	* 321	–	* 2 188	–	1 700	~0.10
1985	136	2 009	0.08	* 347	~0.10	1 662	0.08	1 174	~0.10
1988 a	53	–	–	–	–	1 140	> 0.15	–	–
1988 b	53	–	–	–	–	744	> 0.15	–	–
1991	158	1 447	0.09	686	0.10	761	0.10	197	0.12
1992	191	1 642	0.08	862	0.10	780	0.08	172	0.11
1995	181	2 480	0.07	1 002	0.09	1 478	0.07	317	0.12
1996	193	1 755	0.07	959	0.09	796	0.08	157	0.11
1998	53	2 401	0.18	1 520	0.20	880	0.17	114	0.20
1999	47	3 486	0.12	2 165	0.12	1 321	0.14	194	0.32
2000	50	1 906	0.23	1 336	0.24	570	0.25	89	0.32
2001	51	1 405	0.17	970	0.18	435	0.17	40	0.29
2002	53	1 618	0.14	1 152	0.15	466	0.19	44	0.29

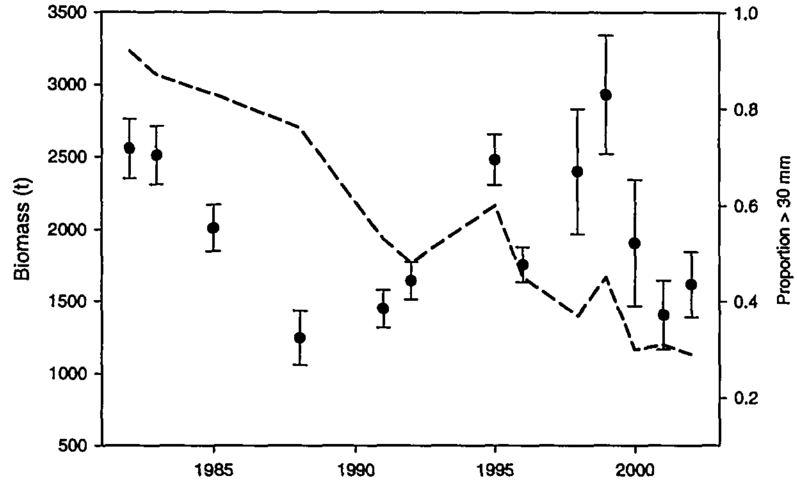

Figure 2: Estimated total biomass of cockles (± one standard error) on Snake Bank, 1982–2002, and the estimated proportion of biomass (dashed line) accounted for by cockles of 30 mm or more shell length.

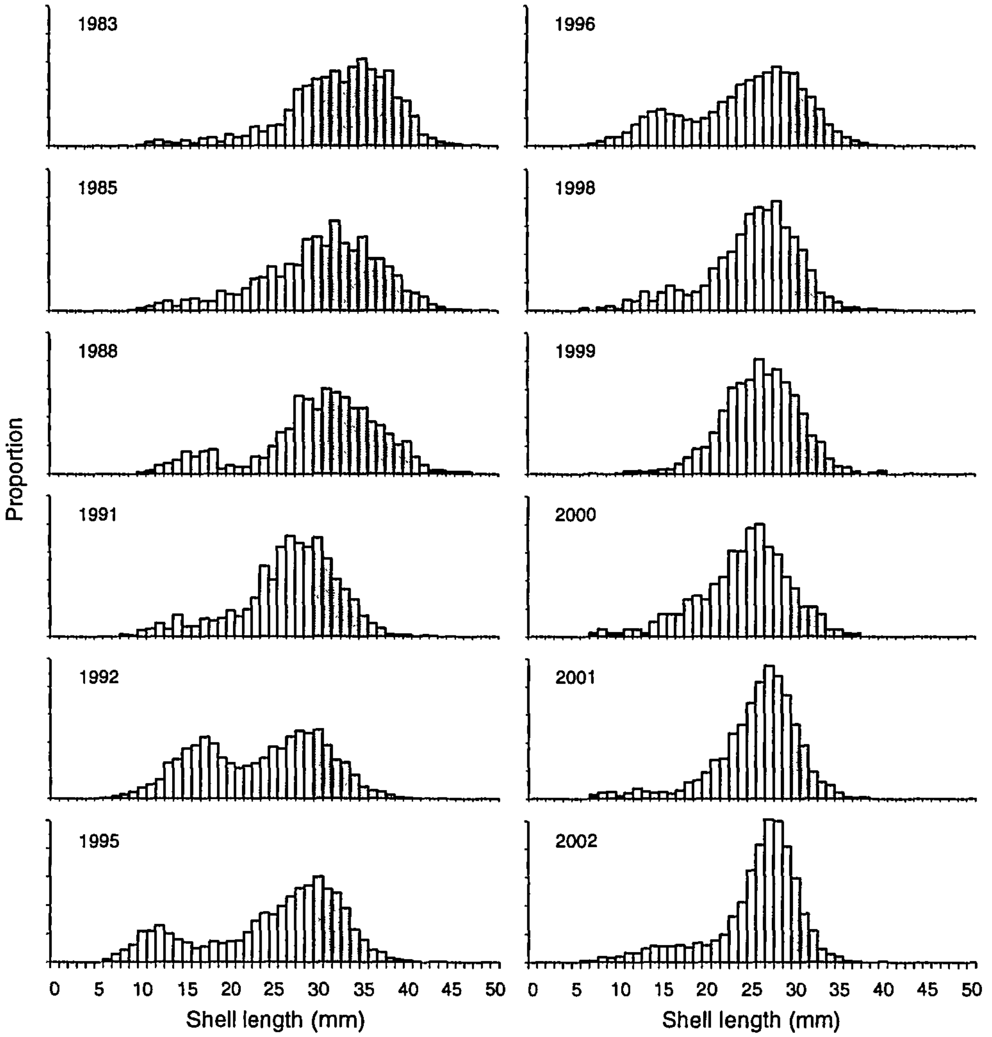

Figure 3: Estimated population length frequency distribution of cockles on Snake Bank since 1983 (data for 1982 lost). The shaded area in each histogram shows those animals 30 mm shell length and larger (the nominal size at recruitment to the fishery).

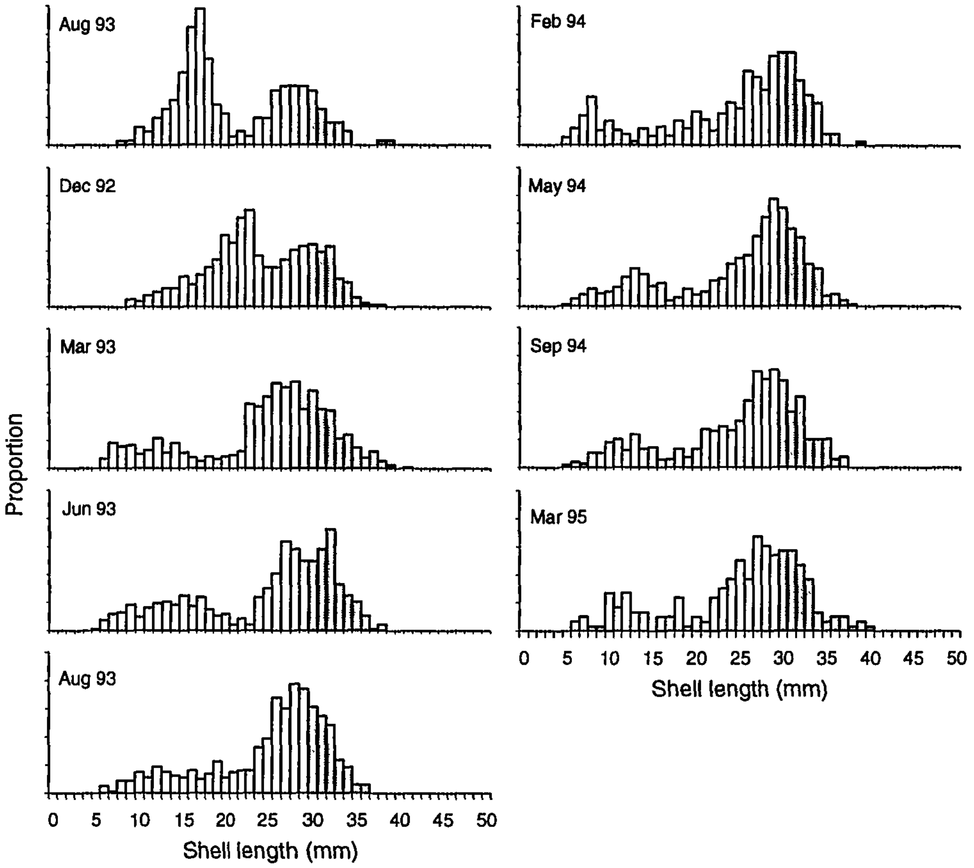

Figure 4: Estimated length frequency distribution of cockles near the lagoon on Snake Bank between 1993 and 1995 (data collected to estimate growth rate. The shaded area in each histogram shows those animals 30 mm shell length and larger (the nominal size at recruitment to the fishery).

9.3 Structure of the model

A length based model was used to assess the snake bank cockle population. This model was adapted from a model developed by Breen et al. (2000) to assess paua (*Haliotus iris*) in PAU 5B and 5D. The model is a stochastic, dynamic, length-based observation-error time series model. It is stochastic because annual variations in recruitment can be estimated as a vector of free parameters. It is dynamic because no equilibrium, other than in the initial length structure, is assumed. Cockles are represented in the model as numbers-at-length rather than numbers-at-age. The error is assumed to be observation error rather than process error.

The model is length-based, with 47 length 'bins', each of 1 mm shell length. The left-hand edge of the first bin is 4 mm; the largest bin is a plus-group representing cockles larger than 50 mm. Sexes are not distinguished. The time step is one season (3 months). The model is implemented in AD Model Builder™ (Otter Research Ltd., http://otter-rsch.com/admodel.htm). AD Model Builder™ incorporates a Markov chain Monte Carlo procedure for the calculation of Bayesian posterior distributions.

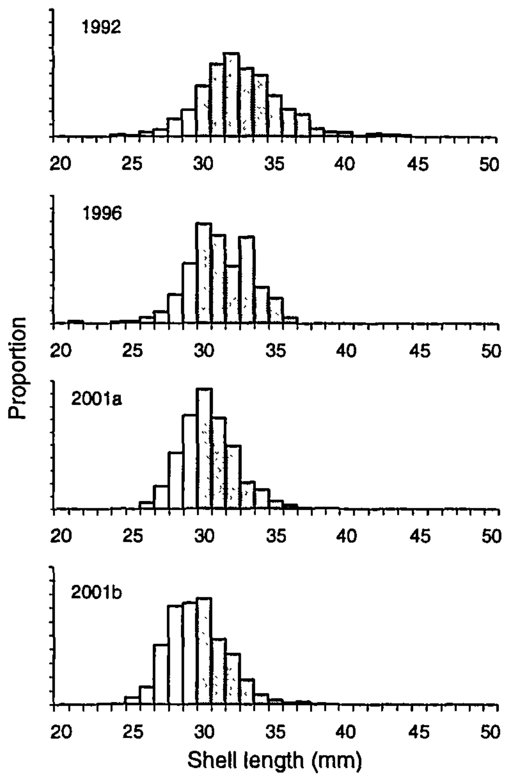

Figure 5: Estimated length frequency distribution of cockles in the commercial harvest from Snake Bank between in 1992, 1996, and 2001 (a = summer, b = winter). The shaded part of each histogram contains animals 30 mm shell length and larger (the nominal size at recruitment to the fishery).

The model population is initialised and then driven by reported catches. The model is fitted, using maximum likelihood methods, to vectors of absolute abundance estimates from surveys, survey length frequency samples, and commercial length frequency samples. Outputs are the past, present, and (potentially) projected states of the stock, estimated using Bayesian methods. Parameters estimated in the model are:

$\ln(R0)$	log of base recruitment $R0$
l^R	mean size of new recruits to population
$\left(\sigma^R\right)^2$	variance of the size distribution of newly recruited animals
M	instantaneous annual natural mortality rate
L_∞	asymptotic length
or $g10$	mean growth increment at 10 mm (alternative linear model)
K	Brody coefficient, year^{-1}
or $g30$	mean growth increment at 30 mm (alternative linear model)
α	c.v. of the expected growth increment
σ_{MIN}	minimum standard deviation of expected growth increment
v^{1r}	shape parameter for the left-hand limb of research survey selectivity
v^{1s}	shape parameter for the left-hand limb for commercial catch
ϕ^{diff}	The average shift in length of ϕ_1^s from period to period.
$\tilde{\sigma}$	common component of observation error
ε_t	vector of recruitment deviations

Related parameters that are fixed in the model (see also Appendix 1) are:

ϕ^r	size of maximum selectivity for research surveys fixed at 11.1 mm
ν^{2r}	shape parameter for the right-hand limb of research survey selectivity fixed at 10^8
ϕ_1^s	size of maximum selectivity for commercial catch; fixed in period 1 at 32 mm
ν^{2s}	shape parameter for the right-hand limb for commercial catch fixed at 200

9.4 Fitting the model and diagnostics

Initial Conditions

The initial population is assumed to be in equilibrium with zero fishing mortality and the base recruitment. The model is run for 60 periods with no fishing to obtain near equilibrium in numbers-at-length. To start, recruitment is evenly divided among the bins (it is added to any animals remaining there after growth from the previous quarter) in proportions determined from:

$$\beta_k^R = \exp\left(-\left(\frac{l_k - l^R}{\sigma^R}\right)^2\right) \tag{5}$$

where l_k is the midpoint length of the kth length class, l^R and σ^R are the mean and the standard deviation of the distribution of recruited cockle's length. The last bin acts as a "plus group". The recruitment happens annually in a chosen season for the burn-in and project periods, and in any period with $\tau_t^R = 1$ for periods with data, hence the number of recruits in period t is:

$$R_{k,t} = \tau_t^R \beta_k^R R0 \tag{6}$$

where the τ_t^R is a switch based on whether the recruitment happens in period t ($\tau_t^R = 1$) or not ($\tau_k^R = 0$).

Growth Transition Matrix

During initialisation, the vector N_t of numbers-at-length is determined from numbers in the previous year, survival, the growth transition matrix G and the vector of recruitment:

$$N_t' = N_{t-1}' \bullet G\exp(-M/4) + R_t \tag{7}$$

where the prime (′) denotes vector transposition and the dot (•) denotes matrix multiplication.

Incremental growth in the model was calculated using either a linear increment (von Bertalanffy) or exponential increment growth function. Variation in growth about the mean increment was determined using either normal or log normal error structure. Three options for describing incremental growth and variation were investigated.

1. *von Bertalanffy function normal error structure*

A growth transition matrix is calculated inside the model from the estimated growth parameters. The expected annual growth increment for the *k*th length class is given by the von Bertalanffy growth function for $l_k < L_\infty$ and is zero where $l_k \geq L_\infty$:

$$\Delta l_k = (L_\infty - l_k)(1 - \exp(-K/4)) \quad \text{for } l_k < L_\infty \qquad (8a)$$

$$\Delta l_k = 0 \quad \text{for } l_k \geq L_\infty \qquad (8b)$$

The standard deviation of this increment is assumed to be proportional to the increment with a minimum σ_{MIN}, and is given by a smooth differentiable function:

$$\sigma^{\Delta l_k} = (\Delta l_k \alpha - \sigma_{MIN})\left(\frac{1}{\pi} \times \tan^{-1}\left((\Delta l_k \alpha - \sigma_{MIN}) \times 10^6\right) + 0.5\right) + \sigma_{MIN} \qquad (9)$$

From the expected increment and standard deviation for each length class, the probability distribution of growth increments for a cockle of length l_k is calculated from the normal distribution, and translated into the vector of probabilities of transition from length bin *k* to other length bins to form the growth transition matrix G. Zero and negative growth increments are permitted.

2. *Linear increment model (von Bertalanffy) log normal error structure*

For many fish species the von Bertalanffy function describes mean length at age well. The rate of change (differential) in mean increment defined by the von Bertalanffy function is linear. A linear increment model was formulated on the basis of two parameters *g10* and *g30* which represent the expected annual growth increment at 10 and 30 mm shell length, respectively.

$$\Delta l_k = \frac{30 g_{10} - 10 g_{30}}{g_{10} - g_{30}} \text{ (after Francis 1988)} \qquad (10)$$

It is preferable to estimate growth relative to these linear parameters and, thereby, avoid the high correlation that typically occurs when *K* and L_∞ parameters are estimated (Francis 1988).

The log-normal probability density for growth increment Δl, for a cockle of length *l* is given by,

$$f(\Delta l_k) = \frac{1}{\sqrt{2\pi}\sigma_k \Delta l} \exp\left[-\tfrac{1}{2}\left(\frac{\log(\Delta l_k) - \log(\Delta l_k') + \tfrac{1}{2}\sigma_k^2}{\sigma_k}\right)^2\right] \qquad (11)$$

where σ_l is the standard deviation of the expected increment at length *l*. Negative growth increments are not permitted.

Dynamics

For each period t, the model calculates the biomass available to the fishery based on the selectivity $V^s_{k,t}$ and the average weight w_k:

$$B_t = \sum_k N_{k,t} V^s_{k,t} w_k \, . \tag{12}$$

The selectivity of the commercial fishery is assumed to have changed over time, taking smaller cockles in later years. Hence, the selectivity, $V^s_{k,t}$ is calculated from:

$$V^s_{k,t} = \lambda^s_{k,t} \exp\left(\frac{(\ln 0.5)(l_k - \phi^s_t)^2}{(v^{1s}_t)^2} \right) + (1 - \lambda^s_{k,t}) \exp\left(\frac{(\ln 0.5)(l_k - \phi^s_t)^2}{(v^{2s}_t)^2} \right) \tag{13}$$

where $\phi^s_t = \phi^s_1 - (t-1)\phi^{diff}$ and $\lambda^s_{k,t} = \dfrac{1}{1 + \exp(-(l_k - \phi^s_t)\delta)}$ (14a & 14b)

where δ is a shape parameter for mixing selectivity curves (assumed to be 5).

The observed catch is then used to calculate exploitation rate, which was limited to a U^{max} with the *posfun* function of AD Model Builder™. If the ratio of catch to biomass exceeds this, then exploitation rate is restricted to just over U^{max} and a penalty is added to the total negative log-likelihood function. Let A_{min} be the survival rate at U^{max}, i.e. $1\text{-}U^{max}$, and A_t be $1\text{-}U_t$:

$$A_t = 1 - \frac{C_t}{B_t} \qquad \text{for } \frac{C_t}{B_t} \le U^{\max} \tag{15}$$

$$A_t = 0.5 A_{\min} \left[1 + \left(1 + 3 - \frac{2\left(1 - \frac{C_t}{B_t}\right)}{A_{\min}} \right)^{-1} \right] \text{for} \frac{C_t}{B_t} > U^{\max} \tag{16}$$

The penalty invoked when exploitation rate is limited is:

$$1000000\left(A_{\min} - \left(1 - \frac{C_t}{B_t}\right)\right)^2 \tag{17}$$

Survival from fishing is calculated as:

$$SF_{k,t} = 1 - (1 - A_t)V^s_{k,t} \tag{18}$$

The vector of numbers-at-length in the following year is calculated from:

$$N_t' = \left(SF_{t-1} \otimes N_{t-1}\right)' \bullet \mathrm{G}\exp(-M/4) + R_t \qquad (19)$$

where ⊗ denotes a vector element product and recruitment is modified by the estimated deviations:

$$R_{k,t} = \tau_t^R \beta_k^R R0 \exp\left(\varepsilon_t - 0.5\sigma_\varepsilon{}^2\right) \qquad (20)$$

Predictions

The predicted survey index is calculated from model numbers in bins greater than 5 mm length, taking into account sampler selectivity-at-length:

$$\hat{J}_t = \sum_{k=2}^{47} N_{k,t} V_k^r \qquad (21)$$

where V_k^r is calculated from:

$$V_k^r = \lambda_k^r \exp\left(\frac{(\ln 0.5)(l_k - \phi^r)^2}{(v^{1r})^2}\right) + (1 - \lambda_k^r)\exp\left(\frac{(\ln 0.5)(l_k - \phi^r)^2}{(v^{2r})^2}\right) \qquad (22)$$

and

$$\lambda_k^r = \frac{1}{1 + \exp\left(-(l_k - \phi^r)\delta\right)} \qquad (23)$$

where δ is assumed to be 5.

The model predicts proportions-at-length for the research survey from numbers in each length class for lengths greater than 5mm:

$$\hat{p}_{k,t}^r = \frac{N_{k,t} V_k^r}{\sum_{k=2}^{47} N_{k,t} V_k^r} \qquad (24)$$

Predicted proportions-at-length for commercial catch sampling are similar starting at length 19mm:

$$\hat{p}_{k,t}^s = \frac{N_{k,t} V_{k,t}^s}{\sum_{k=16}^{k=47} N_{k,t} V_{k,t}^s} \qquad (25)$$

Fitting

The likelihood for the predicted survey index is:

$$L\left(\hat{J}_t \mid \theta\right)=\frac{\varpi^J}{\sigma_t^J \tilde{\sigma}\sqrt{2\pi}}\exp\left[\frac{-\left(\ln(J_t)-\ln\left(\hat{J}_t\right)\right)^2}{2\left(\sigma_t^J\tilde{\sigma}\big/\varpi^J\right)^2}\right] \tag{26}$$

where θ is the parameter vector. The following likelihood is used to fit model predictions to observed proportions-at-length from research surveys (Breen et al. 2002).

$$L\left(\hat{p}_{k,t}^r \mid \theta\right)=\frac{\kappa_t^r\varpi^r\sqrt{\left(p_{k,t}^r+0.1\right)}}{\tilde{\sigma}\sqrt{2\pi}}\exp\left[\frac{-\left(p_{k,t}^r+0.1\right)\left(p_{k,t}^r-\hat{p}_{k,t}^r\right)^2}{2\left(\tilde{\sigma}\big/\kappa_t^r\varpi^r\right)^2}\right] \tag{27}$$

where κ_t^r is the square root of numbers measured in period *t*. The negative log-likelihood is summed for all years with observations and for all length classes.

The likelihood components for commercial catch sampling and the quarterly samples are analogous.

The optimum model fit to the data was deemed to be at the mode of the joint posterior distribution (MPD) i.e., the minimum negative log value of the combined likelihoods and priors. The MPD was estimated derived using the auto differentiation-based minimiser in procedure of ADMB.

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Likelihood weighting

Changing the relative weightings on the likelihood terms was achieved by multiplying the common observational error term ($\tilde{\sigma}$) by an inverse scalar (w_{like_x}).

$$\tilde{\sigma}_{w_{like_x}}=\tilde{\sigma}\left(\frac{1}{w_{like_x}}\right) \tag{28}$$

Priors, bounds and assumptions

Bayesian priors were established for all parameters. With the exception of natural mortality, *M*, and recruitment all were uninformative, incorporated simply as uniform distributions with upper and lower bounds set so wide as not to restrict the estimation unless highly implausible values were explored (Appendix 2)

The prior probability density for M was calculated from the normal distribution (Appendix 3):

$$L(M)=\frac{1}{\sigma_M^2\sqrt{2\pi}}\prod_t \exp\left(-\frac{M_t^2}{2\sigma_M^2}\right) \tag{29}$$

The prior probability density for the vector of estimated recruitment deviations vector, ε, was calculated from the normal distribution:

$$L(\varepsilon)=\frac{1}{\sigma_\varepsilon^2\sqrt{2\pi}}\prod_t \exp\left(-\frac{\varepsilon_t^2}{2\sigma_\varepsilon^2}\right) \tag{30}$$

Biological assumptions

The length-weight relation was taken from Cryer (1997) and was

$$w_k = 1.60\ 10^{-7}\ l_k^{\ 3.29} \tag{31}$$

where l_k is the length in millimetres and w_k is the weight in kilograms.

In calculating spawning biomass, maturity-at-length, m_k, was assumed to be knife-edged at 19 mm. Spawning biomass is:

$$S_t=\sum_k N_{k,t}m_k \tag{32}$$

Bayesian Posteriors

The posterior distributions of key model parameters were characterised using the Hastings-Metropolis Markov Chain Monte Carlo (MCMC) sampling procedures (Hastings 1970). The MCMC procedure was run for one million iterations and was sampled at every 100th iteration. The Geweke (1992) test of convergence was applied to each chain from the MCMC sampling process.

10. Results:

Normal error model fits

Under uniform weighting across all likelihoods the model achieved an excellent fit to the biomass estimates (Figure 6). However, the biomass trajectory from this model was highly erratic, the estimate of natural mortality was implausibly high (on its upper bound of $M = 0.8$, Table 3) and the fit to the survey length frequency distributions was poor (especially for the early years where the model did not reproduce the preponderance of very large cockles, Appendix 3). The fit to the commercial length frequency distributions was very poor and predicted a much wider range of cockles, especially small cockles, in the harvest than observed (Appendix 3). The fit to the seasonal samples was also very poor and the model did not reproduce the observed pattern of recruiting cohorts that are such a dominant feature of this data set. A poor fit to the seasonal samples is not necessarily fatal because these samples were collected from one small part of the bank and there may be spatial variation in spat fall and growth. However, we consider poor fits to length frequency distributions integrated over the whole population and to measurements from the commercial catch to be very serious flaws of the model with "equal" likelihood weightings.

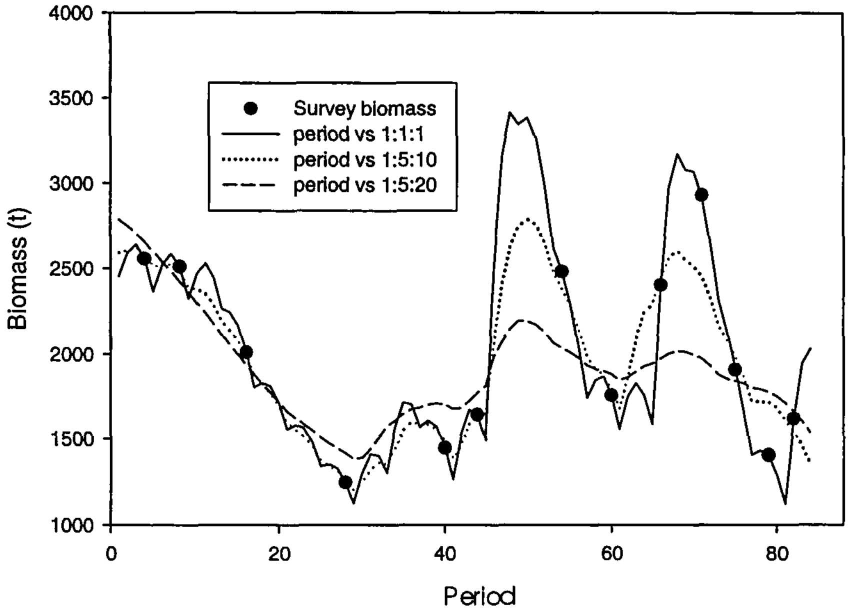

Figure 6: Model fits to survey biomass estimates under increasing weighting toward commercial and survey length frequency data (weighting ratios biomass : commercial LF : survey LF)

Improved fits to length frequency data and more plausible estimates of M were achieved by increasing the relative weighting on the commercial and survey length frequency data (Table 3; Appendix 4). However, these improvements were achieved at the expense of the fit to the biomass series (Figure 6). A weighting factor of 5 and 20 on the commercial and survey length frequency likelihoods, respectively, was thought to give the best overall fit to the observed data and we have nominated this as the base case (Figure 6; Table 3; Appendix 4). As well as fitting the length frequency data better, this fit also has a much less erratic trajectory than that having "equal" weightings. The 1:5:20 model reproduced the commercial and seasonal length frequency distributions reasonably well, but still failed to predict the very large cockles that dominated the population in the early 1980s.

Assuming normal error around the mean growth increment allows negative growth of cockles in the model. In the base case, 38% of animals with starting widths 28 mm or greater shrank at least 1 mm over the annual growth period (Figure 7). We think this amount of negative growth to be biologically implausible, and this was our motivation for testing a lognormal error structure as an alternative.

Normal error model base case diagnostics

Natural mortality (*M*) and mean recruitment (*Rcoff*) were highly correlated in the fitting process (0.983, Appendix 5a) and there was significant correlation between the two growth parameters *K* and *Linf* (Appendix 6a). Most of the annual recruitment parameters were independent of one other.

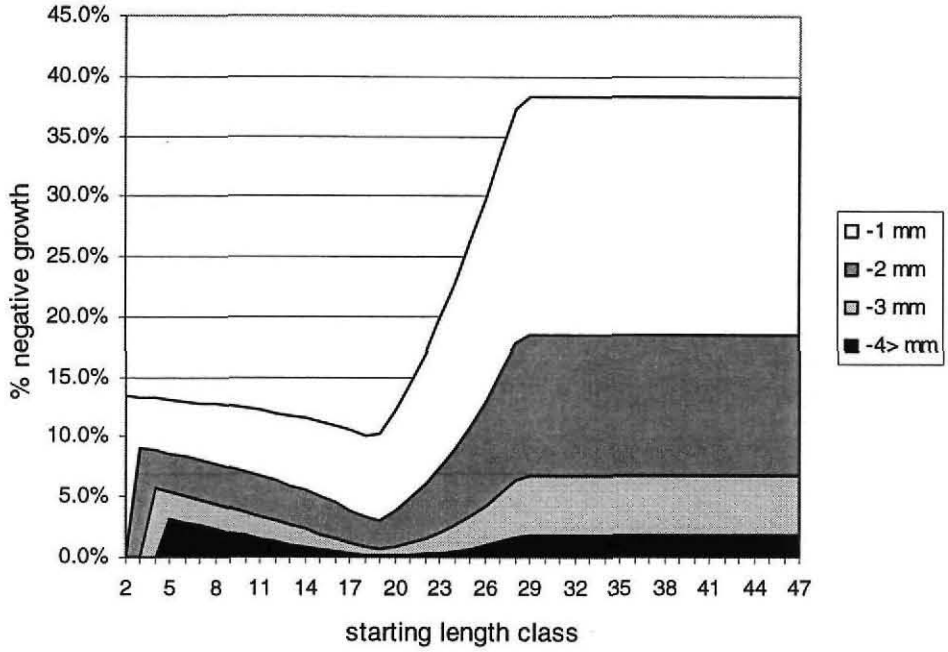

Figure 7: Percentage of animals growing negatively under base case model fit.

Most of the posterior distributions of the main model parameters were unimodal, and the MLE values generally corresponded with the modes of posterior distributions (Appendix 6). Given that most of the model priors were uniform, this pattern in the posteriors would generally suggest that the model was able to find optimum solutions for most parameters. However, there was evidence of a high degree of autocorrelation in the MCMC traces (Appendix 6) and tests indicated that convergence was not attained for several parameters, including some important ones (Appendix 7a). We conclude that, although optimal solutions can be found for most parameters, the model provides a poor description of the observed data. Our confidence that the estimated parameter values have biological meaning is, therefore, low.

Table3: Parameter estimates from Maximum Probability Density (MPD) fits to observational data under various weighting scenarios (weighting ratios biomass : commercial LF : survey LF) [Auto differentiation failed to converge (i.e., the Hessian matrix was not positive definite); shaded denotes parameter value on boundary; bold values denotes "base case"]**

	Normal Error Model Linf & K estimated						Lognormal Error Model g10 & g30 estimated					
Parameters	**1:1:1	1:1:5	1:5:5	1:5:10	1:5:15	1:5:20	1:1:1	1:1:5	**1:5:5	1:5:10	1:5:15	**1:5:20
$ln(R0)$	19.38	19.23	19.29	18.91	18.09	**17.73**	19.01	**19.04**	19.08	18.97	18.40	17.92
l^R	2.44	2.45	2.30	3.16	3.15	**3.17**	3.18	**3.12**	3.14	20.00	16.44	14.36
M	0.80	0.80	0.80	0.53	0.27	**0.20**	0.64	**0.66**	0.66	0.53	0.34	0.24
L_∞ or $g30$	31.63	32.02	31.96	31.78	31.70	**31.74**	0.55	**0.54**	0.50	0.68	0.68	0.65
K or $g10$	1.45	1.40	1.38	1.10	0.85	**0.77**	6.01	**5.91**	5.89	1.90	2.24	2.18
v^{1r}	4.62	3.68	4.06	3.58	3.04	**2.77**	2.43	**1.57**	1.64	49.99	49.99	49.99
v^{1s}	0.00	2.19	1.30	1.57	0.00	**1.59**	0.38	**0.24**	1.06	0.00	0.05	0.44
ϕ^{diff}	-0.58	-0.02	-0.03	-0.03	-0.05	**-0.02**	-0.04	**-0.05**	-0.04	-0.05	-0.05	-0.04
ε 1982	0.09	0.45	0.37	0.14	-0.70	**-1.07**	0.55	**0.51**	0.47	-0.79	-0.84	-0.78
ε 1983	0.10	0.13	0.11	0.00	-0.32	**-0.64**	-0.02	**0.01**	0.02	-0.02	-0.22	-0.58
ε 1984	0.06	0.30	0.21	-0.20	-0.68	**-0.82**	0.14	**0.10**	0.08	-0.17	-0.32	-0.47
ε 1985	-0.51	-0.61	-0.55	-0.54	-0.70	**-0.76**	-0.51	**-0.43**	-0.41	-0.75	-0.75	-0.71
ε 1986	-0.38	-0.25	-0.30	-0.60	-0.94	**-0.89**	-0.40	**-0.39**	-0.41	-0.73	-0.83	-0.78
ε 1987	-0.60	-0.45	-0.51	-0.76	-0.92	**-0.82**	-0.56	**-0.56**	-0.58	-0.87	-0.85	-0.70
ε 1988	-0.75	-0.68	-0.69	-0.74	-0.62	**-0.48**	-0.60	**-0.62**	-0.61	-0.97	-0.78	-0.53
ε 1989	-0.07	-0.03	0.02	0.16	0.85	**1.26**	0.16	**0.06**	0.22	0.73	0.92	1.24
ε 1990	0.20	0.35	0.28	0.30	0.33	**0.30**	0.26	**0.33**	0.21	-0.61	-0.41	-0.19
ε 1991	-0.45	-0.60	-0.66	-0.70	-0.33	**-0.09**	-0.63	**-0.72**	-0.72	-1.19	-0.93	-0.59
ε 1992	0.30	0.37	0.33	0.59	1.14	**1.35**	0.53	**0.50**	0.49	0.26	0.82	1.27
ε 1993	1.11	1.11	1.07	1.01	0.82	**0.61**	0.94	**0.95**	0.94	0.96	0.87	0.25
ε 1994	-0.04	0.01	-0.04	-0.40	-0.58	**-0.34**	0.15	**0.03**	0.01	-0.89	-0.61	0.24
ε 1995	-0.36	-0.37	-0.36	-0.27	0.22	**0.52**	-0.02	**0.01**	0.00	-0.35	0.05	0.25
ε 1996	0.16	0.10	0.05	-0.04	0.31	**0.52**	-0.36	**-0.26**	-0.27	-0.11	0.18	0.46
ε 1997	0.22	0.41	0.39	0.87	1.03	**0.95**	0.74	**0.72**	0.70	0.36	0.72	0.73
ε 1998	1.11	0.89	0.85	0.37	0.13	**0.30**	1.03	**0.86**	0.83	-0.16	-0.06	0.13
ε 1999	-0.30	-0.05	-0.16	-0.32	-0.30	**-0.14**	-1.07	**-0.61**	-0.57	0.46	0.13	0.08
ε 2000	-0.70	-0.69	-0.67	-0.31	0.37	**0.73**	-1.33	**-0.85**	-0.81	-0.80	0.27	0.88
ε 2001	-0.19	-0.26	-0.21	0.08	-0.16	**-0.24**	0.23	**0.38**	0.35	-0.83	-0.55	-0.76
ε 2002	0.73	0.67	0.56	-1.04	-1.22	**-1.02**	0.87	**-1.26**	-1.20	-0.42	-0.99	-0.93
Likelihoods												
Like Bio	-63.29	-37.98	-34.20	13.80	80.80	**90.23**	-62.12	**5.39**	2.97	-10.47	63.27	99.04
Like Survey LF	-2305.71	-2355.60	-2356.15	-2417.18	-2550.98	**-2627.02**	-2117.02	**-2293.78**	-2290.95	-2384.71	-2489.09	-2588.00
Like Com LF	-109.73	-117.88	-235.68	-217.54	-185.95	**-163.21**	-240.52	**-106.14**	-245.42	-215.80	-180.94	-157.33
Mprior	3.77	3.82	3.82	2.43	1.56	**3.02**	3.22	**3.27**	3.30	2.40	-1.81	2.21
RecruitPrior	17.23	16.79	15.49	19.49	30.94	**36.55**	26.80	**22.73**	21.40	29.88	28.34	30.54
Total like	-2457.73	-2490.85	-2606.72	-2599.00	-2623.63	**-2660.43**	-2389.64	**-2368.52**	-2508.71	-2578.70	-2580.23	-2613.53
Indicators												
B0	2628	2304	2384	2658	2842	**2782**	2306	**2328**	2356	3142	2994	2855
B0rec	1104	1471	1307	1543	1575	**1956**	1142	**1104**	1311	1639	1755	1912
B2002rec	1957	1136	1189	956	1104	**1041**	1229	**930**	951	820	979	1039
B2002rec/B0rec	1.77	0.77	0.91	0.62	0.70	**0.53**	1.08	**0.84**	0.73	0.50	0.56	0.54

Lognormal error model fits

Under uniform weighting across all likelihoods, the model achieved a good fit to the biomass estimates (Figure 8). The estimated value of *M* under uniform weighting was much higher than the mean of the prior (0.64 *vs.* 0.30), but was not as high as estimated by the normal error model and was not against its bound (Table 3). The lognormal error model's fit to the survey length frequency data was worse than that achieved by the normal error model under uniform weighting (Appendix 3 *cf* Appendix 8) and the fit was especially poor for the most recent surveys (2001 and 2002). Conversely, the lognormal error model reproduced some of the dominant cohort structure in the seasonal samples better than did the normal error model, and the fits to recent catch samples were good.

Unfortunately, increasing the relative weighting on the commercial and survey length frequency data resulted in a poorer fit to the biomass estimates (Figure 8) and did not markedly improve the fit to the length frequency data (although the 1:5:20 weighted model was almost able to reproduce the population of very large cockles in 1983, Appendix 9). Further, models weighted towards the length frequency data failed to converge, produced parameter values on their respective bounds (Table 3), and sometimes produced unstable, "saw tooth" patterns in the biomass. We selected a lognormal model with a weighting of 5 toward the survey length frequency distributions as the base case (1:1:5; Table 3).

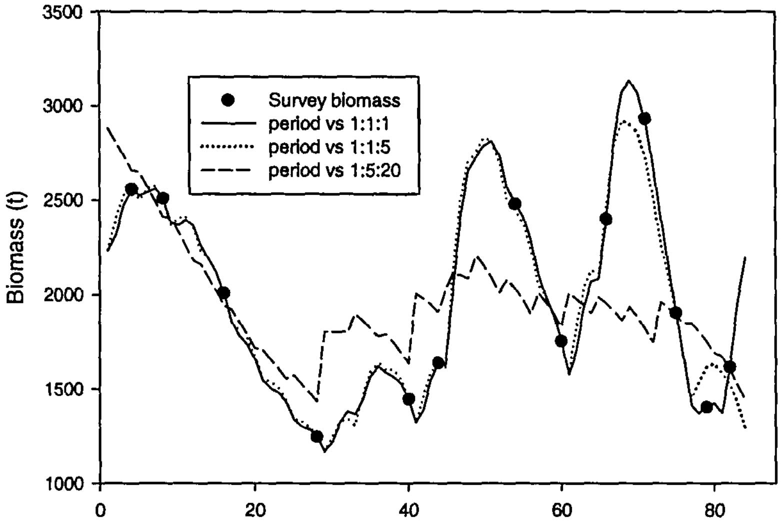

Figure 9: Model fits to survey biomass estimates under increasing weighting toward commercial and survey length frequency data (weighting ratios biomass : commercial LF : survey LF)

Lognormal error model base case diagnostics

As for the normal error model, natural mortality (*M*) and mean recruitment (*Rcoff*) were correlated in the fitting process, though not as badly as in the normal error model (0.607 Appendix 5b). Conversely, mean recruitment was highly correlated with a large proportion of the annual recruitment parameters (Appendix 5b) and there was higher correlation among recruitment parameters than in the normal error model (Appendix 5a).

The posterior distributions for the three selectivity parameters ($v^{1r}, v^{1s}, \phi^{diff}$) were broad, suggesting that these parameters were not well estimated (Appendix 10 f, g, h). There was a high degree of auto correlation in MCMC chains (Appendix 11) and, as with the normal error model, convergence was not attained for several parameters, including some important ones (Appendix 7b). We conclude that this model, too, provides a poor description of the observed data, and our confidence that the estimated parameter values have biological meaning is, therefore, low.

11. Discussion:

11.1 General

Both normal and lognormal error models had problems rationalising the observed biomass and length frequency data; fits were obtained to one series at the expense of the fit to the other. There seems to be a fundamental conflict in the observed data, and this may point to the existence of an "unseen" or unaccounted mortality factor impacting upon the cockle population, or high variability of growth or mortality between years. Landings were relatively low and stable over the period of rapid decline in biomass in the 1980s, whereas reported landings increased rapidly as biomass increased in the early 1990s (Figures 6 & 8). Within both models, the only means of "shedding" or "gaining" the required biomass over such short times are through, respectively, implausibly high rates of natural mortality and improbably fast growth. That the normal error model was slightly better at finding a compromise between the observational data sets is not surprising because it has the capacity to remove biomass from the population rapidly by allowing a large proportion of harvestable cockles to shrink.

Using our base cases models, we estimate the current recruited biomass of cockles on Snake Bank to be slightly over 50% of virgin recruited biomass. However, the apparent incompatibility of data sets, the favouring of implausibly high rates of mortality and growth, and the lack of convergence inferred from MCMC chains are reasonably serious shortcomings of the models, and we have little confidence in any estimates of current stock status.

11.2 Future development of the model

We think that both normal and lognormal error models are over-parameterised; there seems to be insufficient information in the observed data to estimate all the parameters. Both models would benefit from the inclusion of independent data on growth (because growth and mortality are inextricably linked and the model frequently favoured implausible estimates for both). A notch tagging study is underway on Snake Bank and some recaptures have been made. Further tagging and recapture events should provide sufficient marginal increment data to include a new likelihood term for these data in the model. This should help constrain the model to more realistic values for natural mortality as well as growth.

Broad posterior distributions for the selectivity parameters in the lognormal error model suggest that these parameters were not well estimated although the base case lognormal model fits to the commercial length frequency distribution appear much better than those for the normal error model. More data are being collected on the length frequency distribution of the commercial catch in 2003 and this may help.

Since the underlaying model is expressed in terms of numbers at length, we suggest the population likelihood term would be better formulated as population numbers instead of its current formulation based on biomass.

Biomass is calculated in the current model using a length weight relationship without associated error. In fact, the length weight relationships estimated at different times have been different, and there is quite a lot of scatter around each. This may be a source of bias in the estimation and should be investigated.

We have coded an additional term for density dependent growth in the normal error model but because we were exploring other growth models and fits using density-dependent growth took a long time to converge, we have not explored this in detail. The rapid loss of biomass early in the time series and the maintenance of a relatively high biomass despite heavy fishing later in the time series are both intuitively consistent with some density dependent effect. We think this should be explored further in any model development.

Finally, if the reported landings used to drive the model are grossly inaccurate, modelling will be futile unless the actual catches can be estimated. Anecdotes suggest that landings may have been under-reported in some earlier years but, conversely, there is evidence that cockles have been gathered commercially from parts of the harbour other than Snake Bank. Thus, there is the potential for extractions to have been over-reported as well as under-reported. If refining the biological aspects of the model cannot remove the current inconsistencies among data sets and provide for robust parameter estimates, then the potential for mis-reported landings will have to be explored or modelling abandoned (in favour of a simpler "CAY" approach, perhaps).

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11. Conclusions:

1. A stochastic, dynamic, length-based, observation-error, time-series model for cockles on Snake Bank, Whangarei Harbour, is described. The model is driven by reported landings, and fitted using Bayesian techniques to estimates of absolute biomass and length frequency distributions from surveys, to length frequency samples from the commercial catch, and to seasonal length frequency distributions from part of the bank between 1992 and 1995.

2. Two alternative growth models are explored; one has normal error structure and allows negative growth whereas the other has log-normal error structure and does not allow negative growth. Within each error structure, a variety of weighting scenarios were explored, giving more or less relative weight to the different data sets. The selected "base cases" preferentially weight to survey length frequency distributions.

3. Both models seem over-prescribed and tests on MCMC chains suggested the models did not converge on estimates for several parameters, some of them important ones. There was also high correlation among some of the estimated parameters; in the growth parameters in the normal error model, and in the recruitment vectors in the lognormal error model.

4. To explain rapid decreases and increases in observed biomass, both models favour implausibly high rates of natural mortality and growth. Neither model could resolve the apparent incompatibility of the biomass and length frequency data sets, and the results were very sensitive to the relative weights accorded each data set.

5. Both base case models suggest that current recruited biomass is slightly higher than 50% of virgin recruited biomass. However, because neither model provides a reasonable description of the observed data, and may not have converged properly for several parameters, we are not confident that these estimates have biological meaning.

12. Publications:

None.

13. Data Storage:

Length and weight data from surveys are held on a secure, backed-up server at NIWA, Auckland and have been copied to the Empress database, *beach*.

Appendix 1: **Non estimable model parameters.**

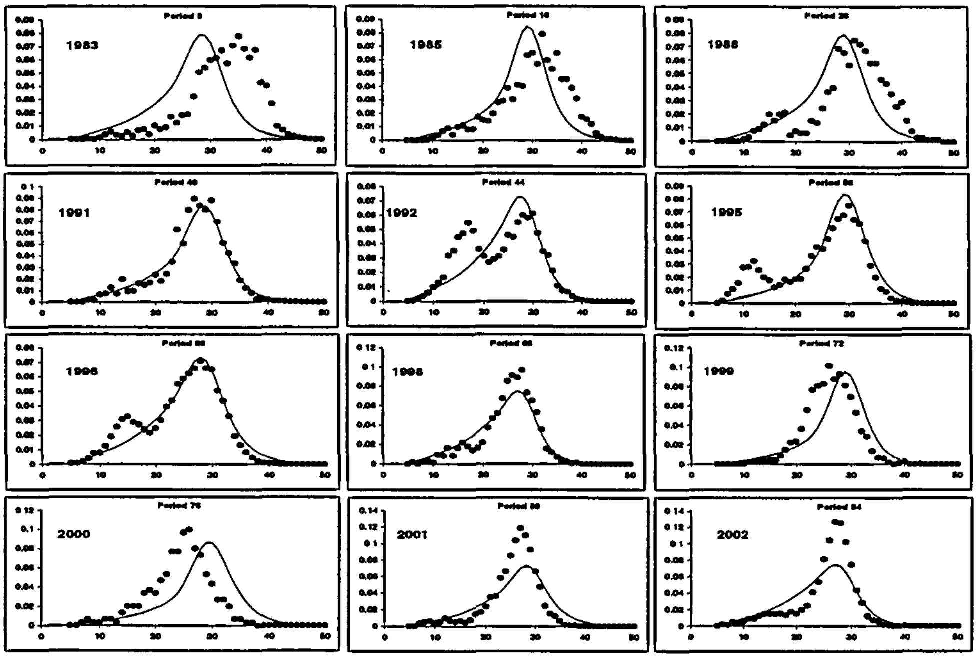
$R0$	base number of annual recruits (= exp(ln($R0$))
β_k^R	proportion of recruits that enter the *k*th length class
τ_t^R	a switch based whether animal is recruited in period t ($\tau_t^R = 1$) or not ($\tau_t^R = 0$)
σ_ε	assumed standard deviation of recruitment deviations in logarithmic space
$N_{k,t}$	number of cockles in the *k*th length class in period t
$R_{k,t}$	recruits to the model in the *k*th length class in period t
Δl_k	expected seasonal growth increment for cockle in the *k*th length class
$\sigma^{\Delta l_k}$	standard deviation of the growth increment for cockle in the *k*th length class
G	growth transition matrix
B_t	biomass of cockles available to the fishery in period t
l_k	length of a cockle at the midpoint of the *k*th length class
w_k	average weight of a cockle at l_k
m_k	proportion of mature cockles at l_k
S_t	biomass of mature cockle in period t
C_t	total observed catch in period t
U_t	exploitation rate in period t
U^{max}	maximum permitted exploitation rate
$SF_{k,t}$	finite rate of survival from fishing for cockles in the *k*th length class in period t
$\hat{J}_t$	predicted research diver survey index
J_t	observed research diver survey index
V_k^r	relative selectivity of research divers for cockles in the *k*th length class
λ_k^r	proportion of mixing the two normal curve for the research survey selectivity
$V_{k,t}^s$	relative selectivity of the commercial fishery for cockles in the *k*th length class, in period t
$\lambda_{k,t}^s$	proportion of mixing two normal curve for the commercial fishery selectivity
ϕ_t^s	size of maximum selectivity for catch sampling selectivity function in period t
σ_t^J	the standard error of the estimate of research survey index in period t
ϖ^J	relative weight assigned to the research diver survey index data set
κ_t^s	a relative weight for length frequency data from commercial catch sampling in period t
$\hat{p}_{k,t}^s$	predicted proportion-at-length in the *k*th length class in period t in commercial catch sampling
$p_{k,t}^s$	observed proportion-at-length in the *k*th length class in period t in commercial catch sampling
ϖ^s	relative weight assigned to the commercial catch sampling length frequency data
κ_t^r	a relative weight for length frequency data from research surveys in period t
$\hat{p}_{k,t}^r$	predicted proportion-at-length in the *k*th length class in period t in research surveys
$p_{k,t}^r$	observed proportion-at-length in the *k*th length class in period t in research surveys
ϖ^r	relative weight assigned to the research survey length frequency data
L	likelihood
μ_M	mean of the prior distribution for M
σ_M	standard deviation of the prior distribution for M

Appendix 2: **Priors and parameter bounds.**

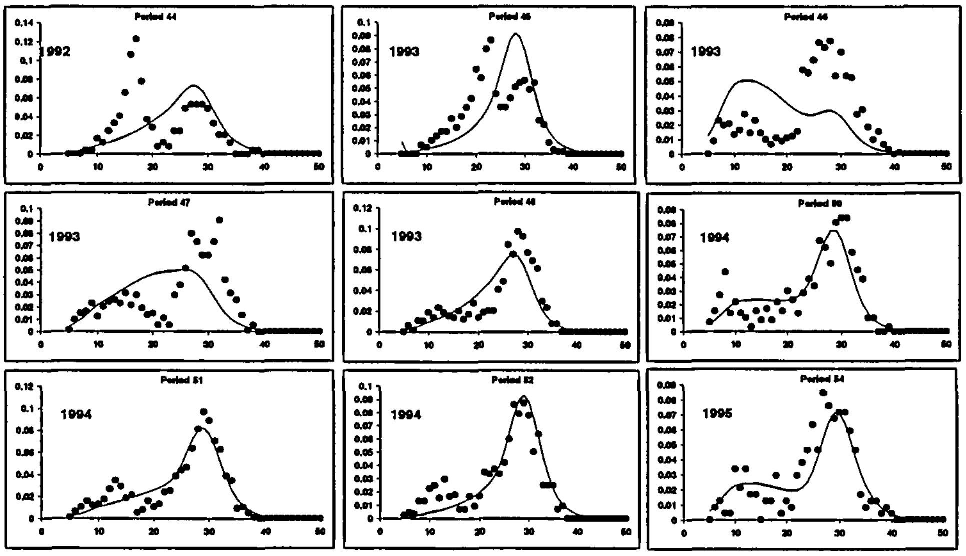
Parameter	prior	mean	cv	lower bound	upper bound
$\ln(R0)$	uniform	-	-	1.000	50.000
l^R	uniform	-	-	2.000	20.000
$(\sigma^R)^2$	uniform	-	-	0.200	100.000
M	normal	0.3	0.8	0.050	0.800
L_∞	uniform	-	-	20.000	50.000
or $g10$	uniform	-	-	1.000	20.000
K	uniform	-	-	0.010	2.000
or $g30$	uniform	-	-	0.000	2.000
α	uniform	-	-	0.001	1.000
σ_{MIN}	uniform	-	-	0.001	5.000
v^{1r}	uniform	-	-	0.001	50.000
v^{1s}	uniform	-	-	0.001	50.000
ϕ^{diff}	uniform	-	-	0.000	1.000
$\tilde{\sigma}$	uniform	-	-	0.010	100.000
ε_t	normal	0.0	0.4	-2.300	2.300

Appendix 3: **Normal error model fits to the survey and commercial length frequency data under equal weighting.**

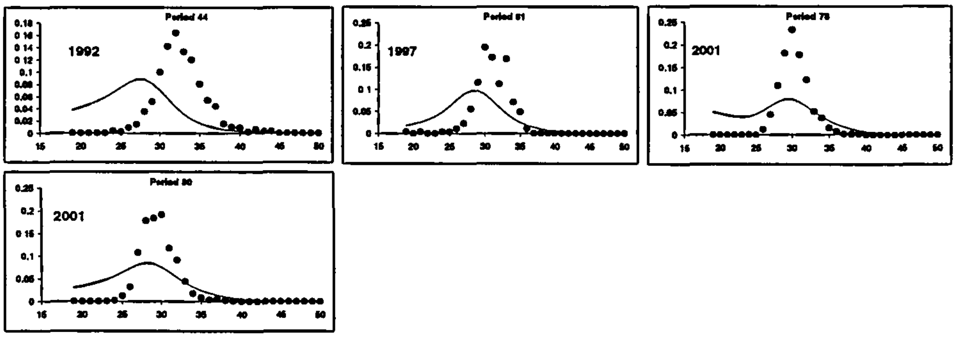
Fits to Survey Length Frequency data 1:1:1



Fits to Seasonal sampling data

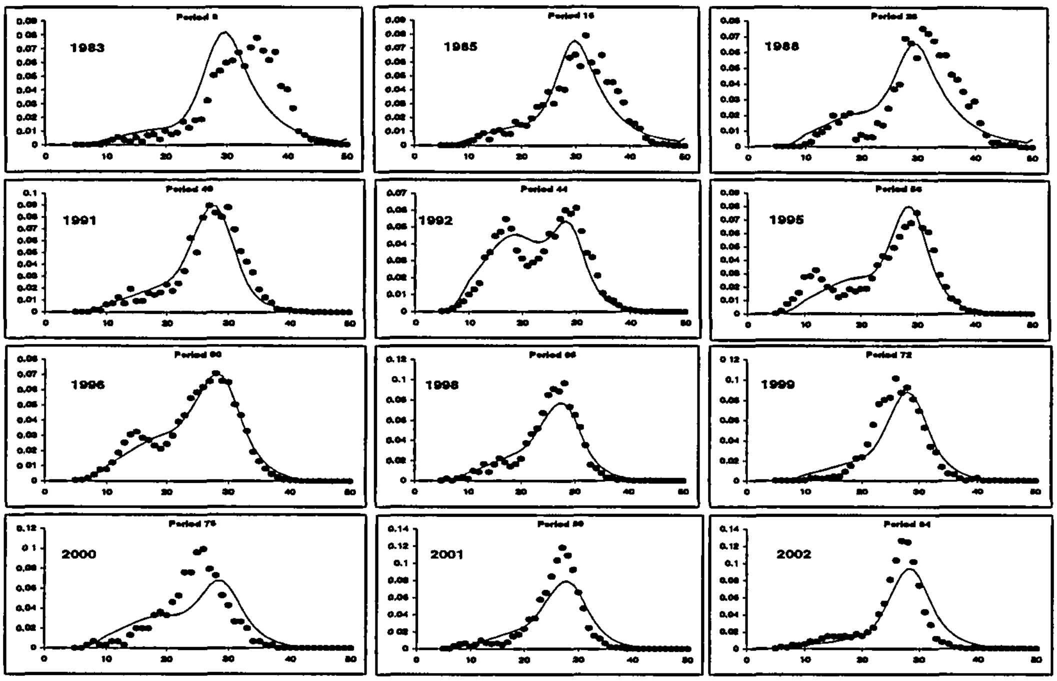


Fits to commercial shed sampling data

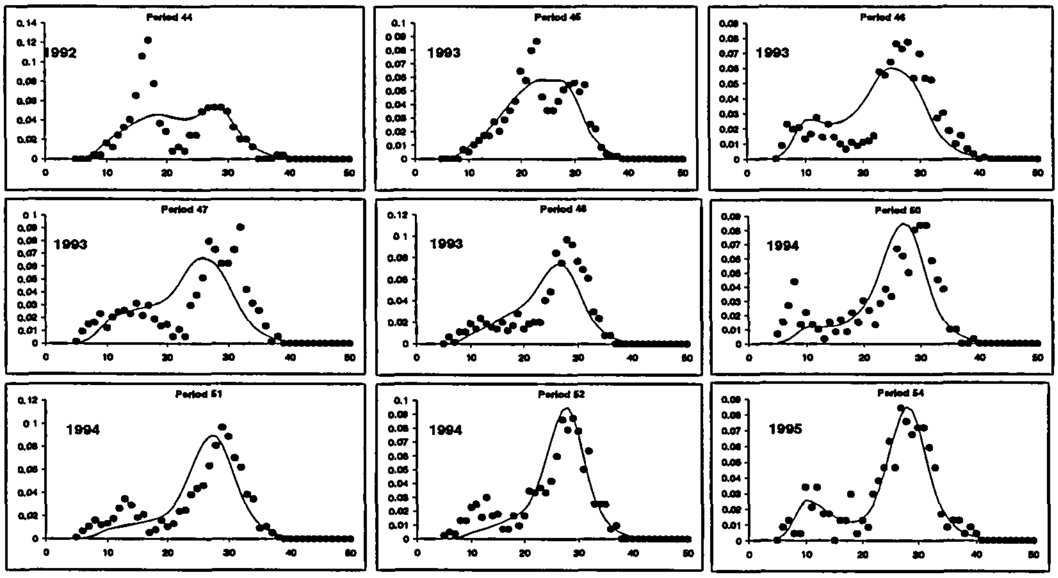


Appendix 4: **Normal error model fits to the survey and commercial length frequency data under high weighting toward the survey length frequency data (*weighting ratios* biomass : com LF : surv LF 1:5:20).**

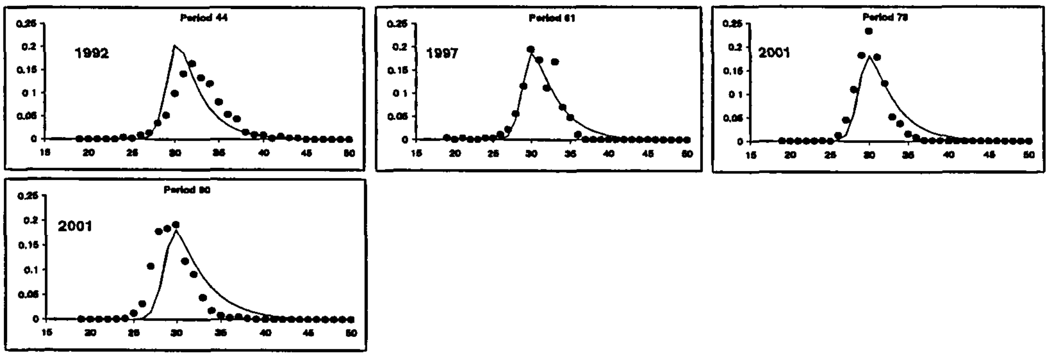
Fits to Survey Length Frequency data 1:5:20



Fits to Seasonal sampling data



Fits to commercial shed sampling data



Appendix 5a: **Normal error model base case (1:5:20) parameter values and correlations from auto-differentiation fitting process.**

	value	CV	Rcoff	sigmatilde	M	Linf	BrodyK	SelectLIS	SelectdiffLF	SelectLLF	RecSize Ex	RecSizeVar	GrowthCV
Rcoff	17.726	0.005	1.000										
sigmatilde	0.327	0.022	0.088	1.000									
M	0.201	0.070	0.983	0.095	1.000								
Linf	31.740	0.005	0.155	0.015	0.139	1 000							
BrodyK	0.774	0.035	0.425	0.058	0.498	-0.634	1.000						
SelectLIS	2.772	0.265	0.028	0.003	0.028	-0.156	0.140	1.000					
SelectdiffLF	0.018	0.387	0.047	0.004	0.067	0.131	-0.030	-0.007	1.000				
SelectLLF	1.588	0.285	-0.034	-0.003	-0.065	-0.163	0.092	-0.006	-0.836	1.000			
RecSize Ex	3.171	74.161	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.000		
RecSizeVar	0.228	115.110	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	-0.760	1.000	
GrowthCV	1.000	0.000	0.002	0.000	0.002	0.001	-0.001	-0.001	0.000	0.000	0.000	0.000	1.000
GStdMin	1.674	0.027	0.017	-0.008	0.108	-0.432	0.443	0.009	0.105	-0.301	0.000	0.000	0.000
Eps 1982	-1.071	-0.243	0.033	0.068	0.096	0.065	0.018	-0.002	0.001	0.006	0.000	0.000	0.000
Eps 1983	-0.641	-0.324	0.238	0.027	0.266	-0.004	0.183	0.019	0.000	0.037	0.000	0.000	0.001
Eps 1984	-0.824	-0.303	0.125	0.038	0.137	0.082	0.029	-0.004	-0.001	0.008	0.000	0.000	0.000
Eps 1985	-0.758	-0.296	0.067	0.032	0.080	-0.048	0.110	0.006	0.005	0.020	0.000	0.000	0.000
Eps 1986	-0.889	-0.306	0.044	0.052	0.050	-0.004	0.035	0.000	-0.001	-0.002	0.000	0.000	0.000
Eps 1987	-0.820	-0.341	-0.011	0.047	-0.008	0.002	-0.008	-0.004	0.000	-0.003	0.000	0.000	0.000
Eps 1988	-0.475	-0.643	-0.062	0.036	-0.063	-0.006	-0.041	-0.012	0.004	-0.005	0.000	0.000	0.000
Eps 1989	1.256	0.117	-0.533	-0.109	-0.551	-0.029	-0.415	-0.056	-0.075	0.064	0.000	0.000	-0.001
Eps 1990	0.303	0.928	0.072	0.038	0.097	0.037	0.124	0.033	-0.006	0.024	0.000	0.000	0.000
Eps 1991	-0.093	-2.199	-0 291	-0.024	-0.312	0.089	-0.311	-0.020	-0.017	0.020	0.000	0.000	-0.002
Eps 1992	1.347	0.060	-0.735	-0.064	-0.727	-0.201	-0.290	-0.006	-0.028	0.013	0.000	0.000	0.000
Eps 1993	0.613	0.233	0.190	0.006	0.217	-0.131	0.238	-0 011	-0.066	0.132	0.000	0.000	0.000
Eps 1994	-0.345	-0.651	-0.184	0.017	-0.183	0.171	-0.210	0.010	-0.003	0.028	0.000	0.000	-0.001
Eps 1995	0.522	0.161	-0.667	-0.063	-0.669	0.114	-0.444	-0.029	-0.043	0.059	0.000	0.000	-0.001
Eps 1996	0.517	0.149	-0.701	-0.042	-0.663	-0.383	0.017	-0.035	-0.059	0.051	0.000	0.000	0.000
Eps 1997	0.947	0.114	-0.155	-0.035	-0.168	0.088	-0.325	0.013	0.019	0.024	0.000	0.000	-0.001
Eps 1998	0.305	0.512	-0.257	-0.012	-0.237	-0.105	0.040	-0.017	-0.035	0.063	0.000	0.000	0.000
Eps 1999	-0.139	-1.283	-0.148	0.010	-0.144	-0.088	-0.002	-0.012	-0.043	0.067	0.000	0.000	0.000
Eps 2000	0.729	0.160	-0.457	-0.050	-0.450	0.053	-0.274	-0.031	-0.012	0.053	0.000	0.000	0.000
Eps 2001	-0.243	-0.910	0.096	0.035	0.136	-0.087	0.341	0.016	-0.031	0.085	0.000	0.000	0.001
Eps 2002	-1.025	-0.230	-0.177	0.046	-0.169	-0.077	0.004	-0.021	-0.061	0.058	0.000	0.000	0.000

	Eps 1982	Eps 1983	Eps 1984	Eps 1985	Eps 1986	Eps 1987	Eps 1988	Eps 1989	Eps 1990	Eps 1991	Eps 1992
Eps 1982	1.000										
Eps 1983	-0.159	1.000									
Eps 1984	-0.021	-0.171	1.000								
Eps 1985	0.038	0.028	-0.224	1.000							
Eps 1986	0.001	-0.030	-0.059	-0.102	1.000						
Eps 1987	-0.001	-0.036	-0.052	-0.085	-0.125	1.000					
Eps 1988	-0.003	-0.034	-0.036	-0.043	-0.057	-0.068	1.000				
Eps 1989	-0.018	-0.162	-0.099	-0.119	-0.144	-0.167	-0.344	1 000			
Eps 1990	0.034	0 050	0.036	0.049	0.059	0.072	0.116	-0.714	1.000		
Eps 1991	0.002	-0.077	-0.038	-0.044	-0.046	-0.029	-0.021	0.405	-0.532	1.000	
Eps 1992	-0.005	-0.157	-0.095	-0.039	-0.037	0.005	0.045	0.343	0.013	0.069	1.000
Eps 1993	0.053	0.076	0.034	0.035	0.033	0.016	0.003	-0.066	-0.081	-0.143	-0.562
Eps 1994	0.017	-0.043	-0.007	-0.021	-0.013	0.000	0.008	0.100	0.024	0.096	0.193
Eps 1995	0.020	-0.131	-0.059	-0.032	-0.032	0.011	0.050	0.361	-0.042	0.251	0.492
Eps 1996	-0.002	-0.110	-0.090	0.005	-0.022	0.012	0.048	0.293	-0.029	0.141	0.515
Eps 1997	0.034	-0.060	-0.014	-0.047	-0.009	0.002	0.009	0.199	-0.060	0.101	0.120
Eps 1998	0.014	-0.019	-0.024	0.017	-0.008	0.005	0.018	0.080	0.014	0.053	0.192
Eps 1999	0.015	-0.010	-0.015	0.009	-0.004	0.004	0.013	0.070	-0.005	0.043	0.123
Eps 2000	0.013	-0.095	-0.043	-0.027	-0.021	0.006	0.030	0.251	-0.024	0.156	0.328
Eps 2001	0.039	0.086	0.026	0.057	0.012	0.000	-0.009	-0.167	0.086	-0.088	-0.077
Eps 2002	0.004	-0.021	-0.018	0.008	-0.003	0.006	0.015	0.066	0.003	0.044	0.139

	Eps 1994	Eps 1995	Eps 1996	Eps 1997	Eps 1998	Eps 1999	Eps 2000	Eps 2001	Eps 2002
Eps 1994	1.000								
Eps 1995	-0.149	1.000							
Eps 1996	0.215	0.371	1.000						
Eps 1997	-0.016	-0.016	-0.236	1.000					
Eps 1998	0.062	0.224	0.322	-0.542	1.000				
Eps 1999	0.011	0.093	0.122	0.105	-0.326	1.000			
Eps 2000	0.117	0.350	0.294	0.009	0.149	-0.263	1.000		
Eps 2001	-0.024	-0.051	0.073	-0.156	0.095	0.127	-0.304	1.000	
Eps 2002	0.026	0.130	0.176	-0.029	0.093	0.047	0.218	-0.114	1.000

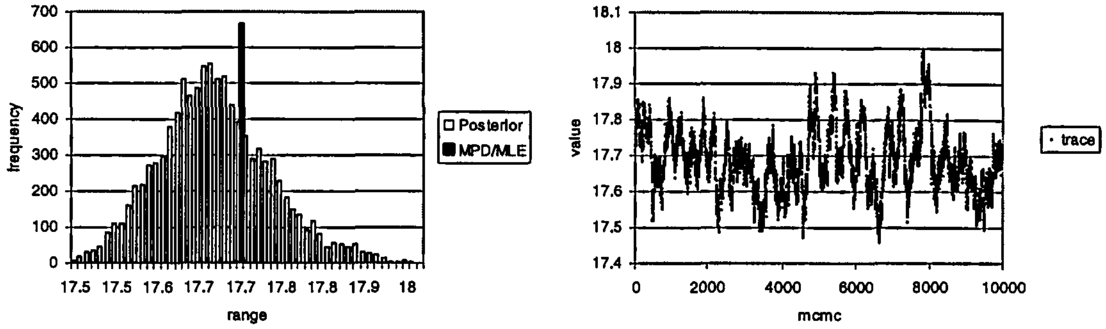
Appendix 5b: Lognormal error model base case (1:1:5) parameter values and correlations from auto-differentiation fitting process.

	value	CV	Rcoff	sigmatilde	M	g30	g10	SelectLIS	SelectdiffLF	SelectLLF	RecSize Ex	RecSizeVar	GrowthCV
Rcoff	19.043	0.004	1.000										
sigmatilde	0.107	0.021	0.018	1.000									
M	0.657	0.040	0.607	0.025	1.000								
g30	0.539	0.046	0.094	-0.002	0.262	1.000							
g10	5.911	0.024	0.065	0.027	0.448	-0.046	1.000						
SelectLIS	1.571	0.559	-0.040	0.000	-0.066	-0.081	-0.005	1.000					
SelectdiffLF	0.047	0.037	-0.003	-0.001	-0.009	0.009	-0.001	-0.002	1.000				
SelectLLF	0.238	0.451	0.011	0.002	-0.018	-0.103	0.003	0.000	-0.357	1.000			
RecSize Ex	3.117	73.055	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.000		
RecSizeVar	0.226	107.842	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	-0.755	1.000	
GrowthCV	1.000	0.000	0.000	0.000	0.001	-0.002	0.002	0.000	0.000	0.000	0.000	0.000	1.000
GStdMin	3.862	0.086	-0 484	-0.012	-0.446	-0.179	0.388	0.001	-0.024	-0.079	0.000	0.000	0.001
Epe 1982	0.510	0.336	-0.740	-0.008	0.020	0.084	-0.002	-0.006	-0.004	-0.023	0.000	0.000	0.000
Epe 1983	0.008	7.062	0.660	0.018	0.292	-0.061	0.301	-0.003	0.002	0.020	0.000	0.000	0.001
Epe 1984	0.104	1.266	-0.386	-0.015	0.176	0.145	0.076	-0.019	-0.010	-0.027	0.000	0.000	0.000
Epe 1985	-0.432	-0.451	-0.185	0.019	0.053	-0.054	0.079	0.005	0.007	0.010	0.000	0.000	0.000
Epe 1986	-0.390	-0.733	-0.087	-0.008	0.096	0.047	0.051	-0.007	-0.002	-0.004	0.000	0.000	0.000
Epe 1987	-0.564	-0.441	-0.147	-0.004	0.083	0.049	0.047	-0.006	-0.002	-0.006	0.000	0.000	0.000
Epe 1988	-0.624	-0.414	-0.173	0.012	0.003	0.001	0.005	-0.001	-0.002	-0.006	0.000	0.000	0.000
Epe 1989	0.060	4.641	-0.196	0.003	-0.127	-0.132	-0.041	0.010	-0.013	-0.001	0.000	0.000	0.000
Epe 1990	0.332	0.503	-0.285	-0.022	0.076	0.136	0.056	-0.001	0.007	-0.012	0.000	0.000	0.000
Epe 1991	-0.722	-0.247	-0 204	0.036	-0.005	0.038	-0.064	-0.023	0.008	0.006	0.000	0.000	-0.001
Epe 1992	0.498	0.145	-0.814	-0.028	-0.317	-0.010	-0.196	0.023	-0.004	-0.016	0.000	0.000	-0.001
Epe 1993	0.950	0.098	-0.487	-0.015	-0.033	-0.078	-0.049	-0.011	0.051	0.006	0.000	0.000	0.000
Epe 1994	0.030	5.445	-0.260	0.014	0.107	0.156	0.170	-0.024	-0.051	-0.018	0.000	0.000	0.000
Epe 1995	0.007	11.040	-0.393	-0.009	0.237	0.130	-0.003	0.016	-0.003	-0.009	0.000	0.000	0.000
Epe 1996	-0.263	-0.285	-0.607	0.008	0.044	-0.045	0.257	0.001	0.027	-0.006	0.000	0.000	0.001
Epe 1997	0.716	0.120	-0.729	-0.017	-0.421	0.012	-0.395	-0.047	-0.011	-0.013	0.000	0.000	-0.001
Epe 1998	0.855	0.097	-0.511	-0.012	0.149	0.061	0.200	0.024	0.020	-0.006	0.000	0.000	0.001
Epe 1999	-0.614	-0.299	-0.035	0.025	0.336	0.044	0.252	-0.017	-0.007	-0.004	0.000	0.000	0.000
Epe 2000	-0.850	-0.184	-0.279	0.019	-0.051	0.009	-0.110	-0.007	0.009	0.000	0.000	0.000	0.000
Epe 2001	0.384	0.192	-0.659	-0.033	-0.041	0.101	-0.043	-0.017	0.003	-0.015	0.000	0.000	0.000
Epe 2002	-1.261	-0.174	-0.110	0.077	0.120	-0.011	0.229	0.016	0.002	0.010	0.000	0.000	0.001

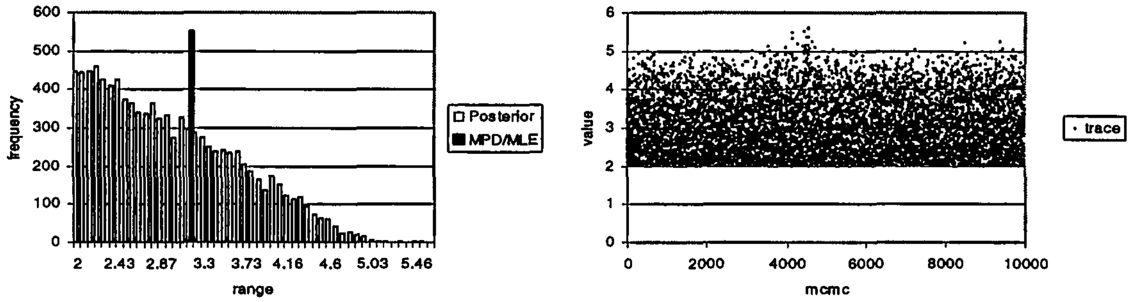
	Epe 1982	Epe 1983	Epe 1984	Epe 1985	Epe 1986	Epe 1987	Epe 1988	Epe 1989	Epe 1990	Epe 1991	Epe 1992
Epe 1982	1.000										
Epe 1983	-0.725	1.000									
Epe 1984	0.653	-0.475	1.000								
Epe 1985	0.275	-0.155	-0.491	1.000							
Epe 1986	0.189	-0.100	0.181	-0.005	1.000						
Epe 1987	0.257	-0.150	0.230	0.004	-0.496	1.000					
Epe 1988	0.230	-0.156	0.179	0.022	-0.299	-0.404	1.000				
Epe 1989	0.154	-0.138	0.073	0.048	0.070	0.062	-0.068	1 000			
Epe 1990	0.422	-0.269	0.288	0.119	0.103	0.128	0.062	-0.759	1.000		
Epe 1991	0.287	-0 211	0.178	0.065	0.051	0.072	0.067	0.123	-0.240	1.000	
Epe 1992	0.829	-0.682	0.488	0.207	0.122	0.184	0.203	0.289	0.315	-0.065	1.000
Epe 1993	0.836	-0.449	0.394	0.192	0.115	0.159	0.148	0.100	0.259	0.231	0.509
Epe 1994	0.374	-0.211	0.276	0.095	0.084	0.108	0.085	0.040	0.180	0.086	0.280
Epe 1995	0.750	-0.468	0.524	0.211	0.162	0.210	0.173	0.087	0.329	0.219	0.558
Epe 1996	0.759	-0 456	0.479	0.261	0.147	0.200	0.181	0.113	0.332	0.222	0.604
Epe 1997	0.684	-0.647	0.378	0.149	0.088	0.140	0.157	0.158	0.256	0.216	0.728
Epe 1998	0.745	-0.444	0.512	0.217	0.157	0.206	0.173	0.105	0.329	0.197	0.571
Epe 1999	0.290	-0.072	0.230	0.123	0.085	0.101	0.069	0.002	0.147	0.085	0.122
Epe 2000	0.357	-0.279	0.220	0.094	0.061	0.086	0.083	0.063	0.143	0.112	0.321
Epe 2001	0.838	-0.601	0.541	0.222	0.155	0.213	0.193	0.133	0.356	0.242	0.720
Epe 2002	0.184	-0.051	0.129	0.086	0.046	0.058	0.046	0.013	0.092	0.051	0.101

	Epe 1994	Epe 1995	Epe 1996	Epe 1997	Epe 1998	Epe 1999	Epe 2000	Epe 2001	Epe 2002
Epe 1994	1.000								
Epe 1995	-0.092	1.000							
Epe 1996	0.364	0.355	1.000						
Epe 1997	0.184	0.454	0.398	1.000					
Epe 1998	0.321	0.575	0.611	0.164	1.000				
Epe 1999	0.138	0.279	0.296	0.178	-0.119	1.000			
Epe 2000	0.111	0.267	0.251	0.265	0.341	-0.325	1.000		
Epe 2001	0.315	0.621	0.626	0.612	0.616	0.223	-0.029	1.000	
Epe 2002	0.095	0.143	0.227	0.027	0.178	0.139	0.105	-0.006	1.000

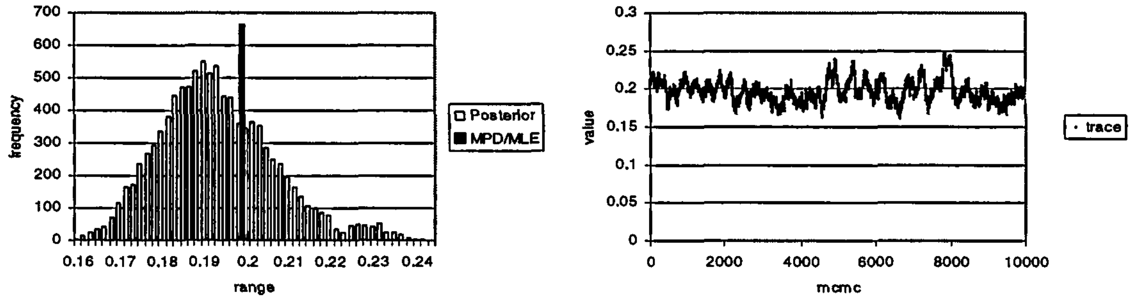
Appendix 6: Normal model posterior distributions and traces (chain length 10 000 sampled from 1 000 000 MCMC steps)



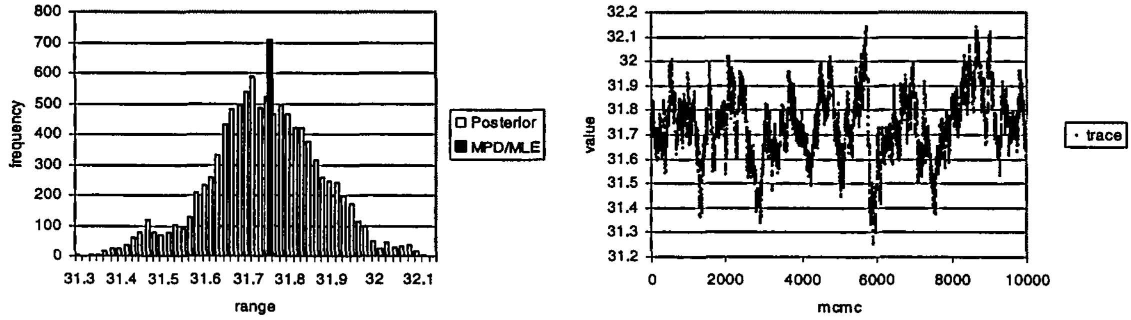
a. Log of recruitment *ln(R0)*



b. Length at first recruitment l^R

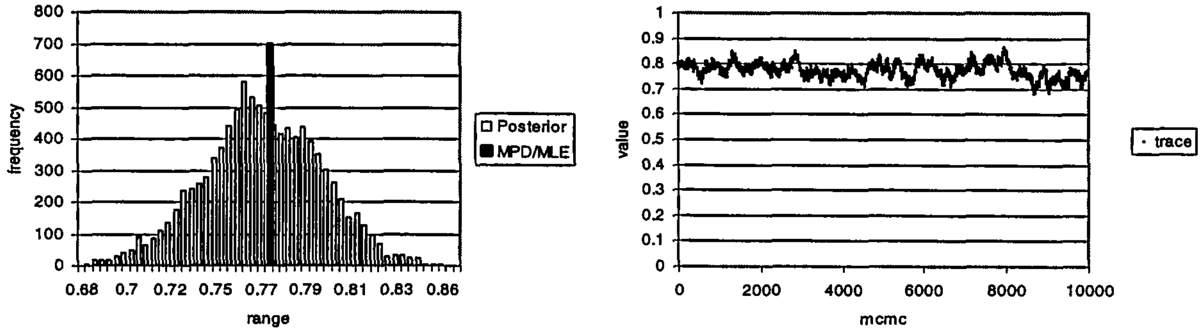


c. Natural Mortality *M*

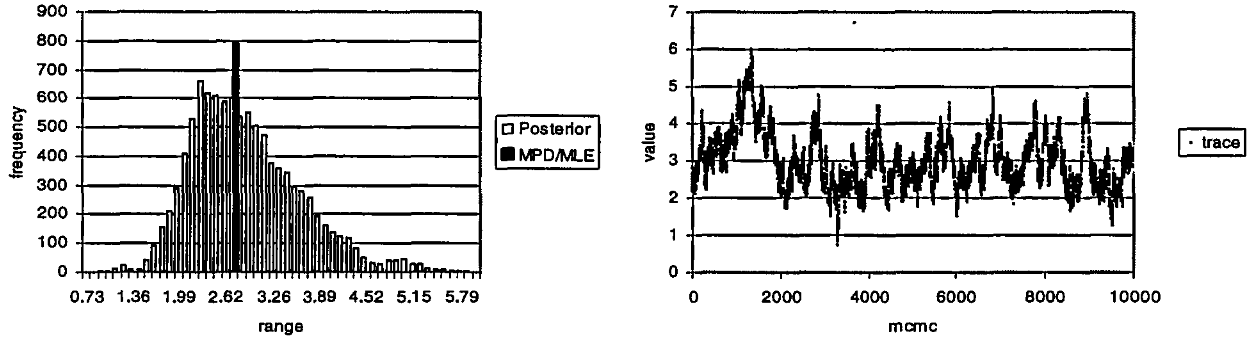


d. Brody L infinity L_∞

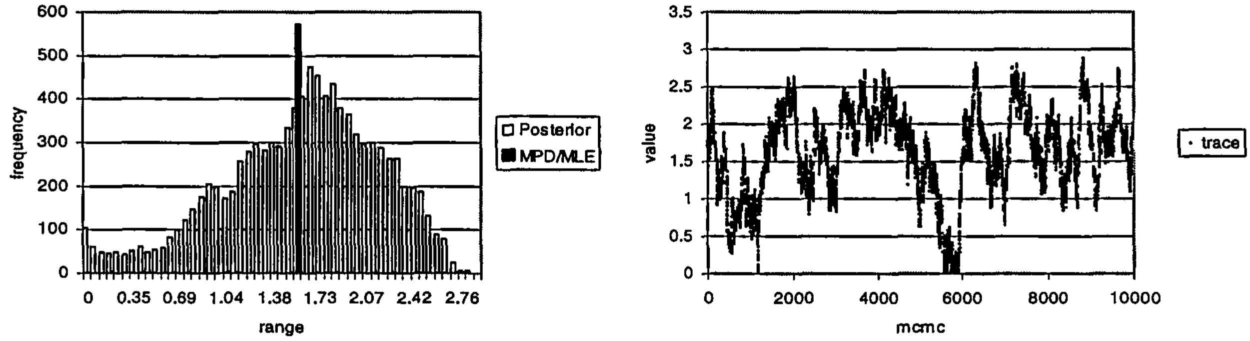
Appendix 6 cont:



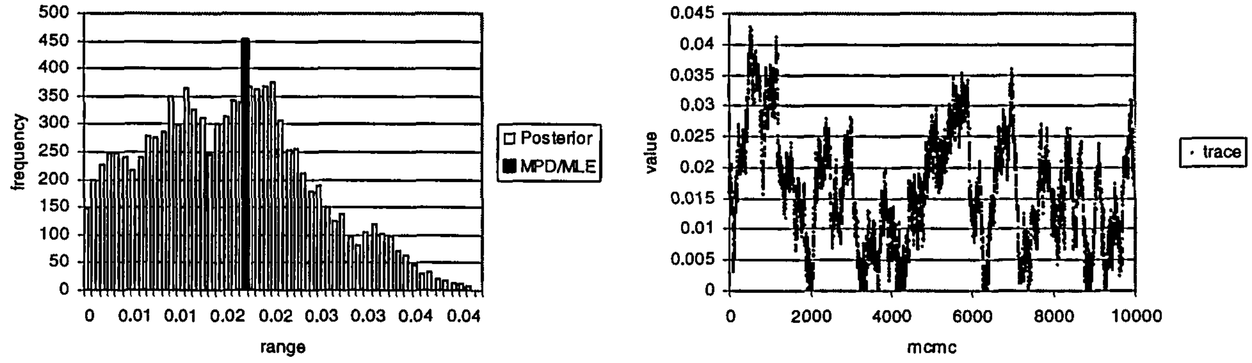
e. Brody K



f. shape parameter for the left hand limb of research survey selectivity ν^{1r}



g. shape parameter for the left hand limb commercial survey selectivity ν^{1s}



h. Annual difference in commercial selectivity mean ϕ^{diff}

Appendix 7: **Geweke convergence diagnostics for MCMC chains. The P-values were generated using two tail Z-tests of the null hypothesis (H_0) that the first 10% of the chain and the last 50% of the chain come from the same population. It is common practice to conclude that there is evidence against convergence when $P < 0.05$.**

a. Normal error model base case

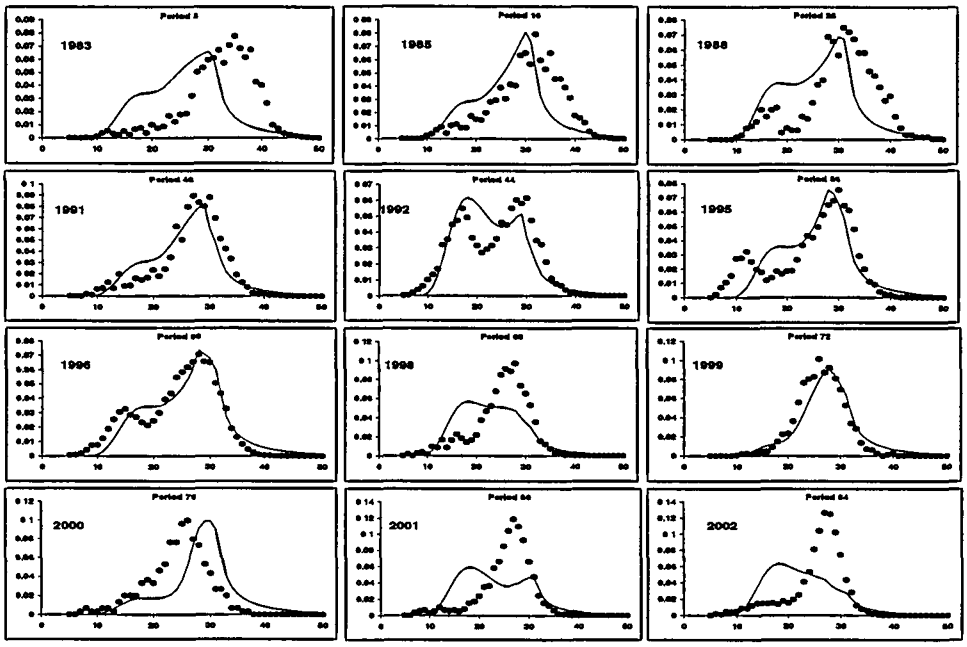
Parameter	Z-Score	p-value	pass/fail
ln(R0)	3.461501	0.001	fail
l^R	-0.644644	0.519	pass
M	5.084667	0.000	fail
L_∞ *or* $g30$	0.147982	0.882	pass
K *or* $g10$	5.818667	0.000	fail
v^{1r}	-6.766396	0.000	fail
v^{1s}	5.674292	0.000	fail
φ^{sr}	9.94504	0.000	fail
ε 1982	8.607849	0.000	fail
ε 1983	3.088832	0.002	fail
ε 1984	-2.311027	0.021	fail
ε 1985	-10.22158	0.000	fail
ε 1986	7.360203	0.000	fail
ε 1987	8.584229	0.000	fail
ε 1988	-0.443228	0.658	pass
ε 1989	-3.691601	0.000	fail
ε 1990	0.993183	0.321	pass
ε 1991	-3.327398	0.001	fail
ε 1992	-5.932129	0.000	fail
ε 1993	6.295206	0.000	fail
ε 1994	-0.650703	0.515	pass
ε 1995	-2.022365	0.043	fail
ε 1996	-3.077236	0.002	fail
ε 1997	-5.132583	0.000	fail
ε 1998	-7.25622	0.000	fail
ε 1999	10.96383	0.000	fail
ε 2000	-9.377436	0.000	fail
ε 2001	5.344583	0.000	fail
ε 2002	-10.81582	0.000	fail
B0	2.221315	0.026	fail
B0rec	-5.420128	0.000	fail
B2002rec	2.262185	0.024	fail
B2002rec/B0rec	4.72718	0.000	fail

b. Lognormal error model base case

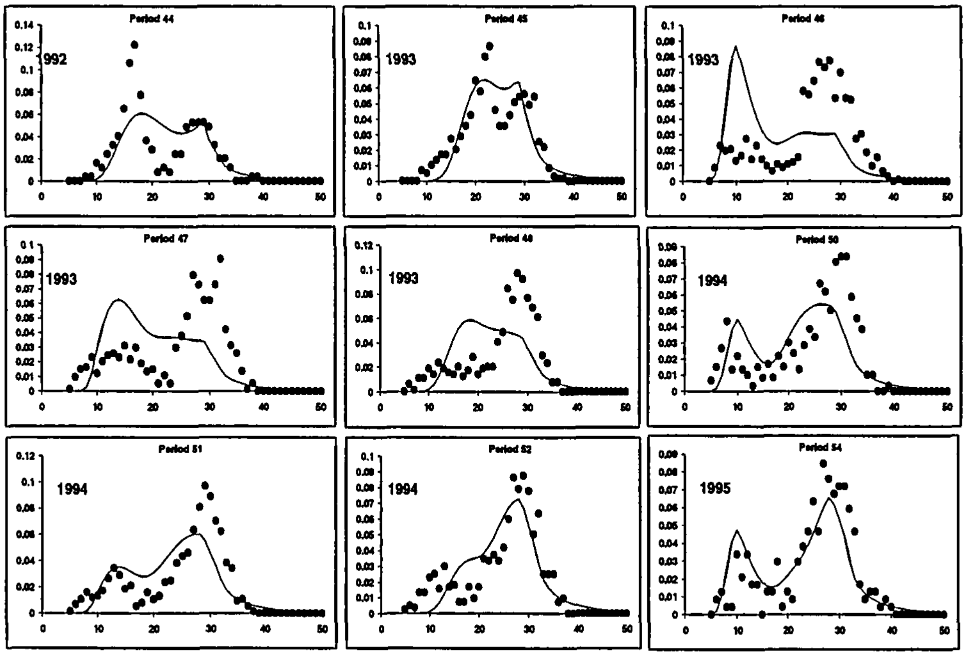
Parameter	Z-Score	p-value	pass/fail
ln(R0)	17.79603	0.000	fail
l^R	-1.430893	0.152	pass
M	13.20311	0.000	fail
$g30$	0.798906	0.424	pass
$g10$	1.771632	0.076	fail
v^{1r}	6.256203	0.000	fail
v^{1s}	-29.98975	0.000	fail
φ^{sr}	36.7378	0.000	fail
ε 1982	-9.855305	0.000	fail
ε 1983	4.51492	0.000	fail
ε 1984	0.02641	0.979	pass
ε 1985	-10.44307	0.000	fail
ε 1986	6.697309	0.000	fail
ε 1987	11.42401	0.000	fail
ε 1988	-16.4652	0.000	fail
ε 1989	1.697791	0.090	pass
ε 1990	-5.082495	0.000	fail
ε 1991	-6.898729	0.000	fail
ε 1992	-11.70173	0.000	fail
ε 1993	-7.161302	0.000	fail
ε 1994	-4.996716	0.000	fail
ε 1995	-3.424094	0.001	fail
ε 1996	-13.76289	0.000	fail
ε 1997	-14.17461	0.000	fail
ε 1998	-5.43286	0.000	fail
ε 1999	5.623511	0.000	fail
ε 2000	-17.41529	0.000	fail
ε 2001	-2.424601	0.015	fail
ε 2002	-10.09547	0.000	fail
B0	9.756857	0.000	fail
B0rec	-11.17416	0.000	fail
B2002rec	5.061952	0.000	fail
B2002rec/B0rec	10.69674	0.000	fail

Appendix 8: Lognormal error model fits to the survey and commercial length frequency data under equal weighting.

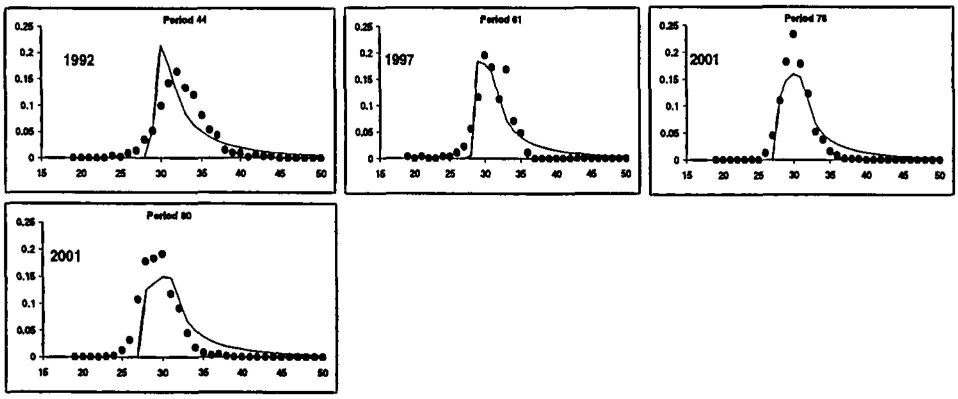
Fits to Survey Length Frequency data 1:1:1



Fits to Seasonal sampling data

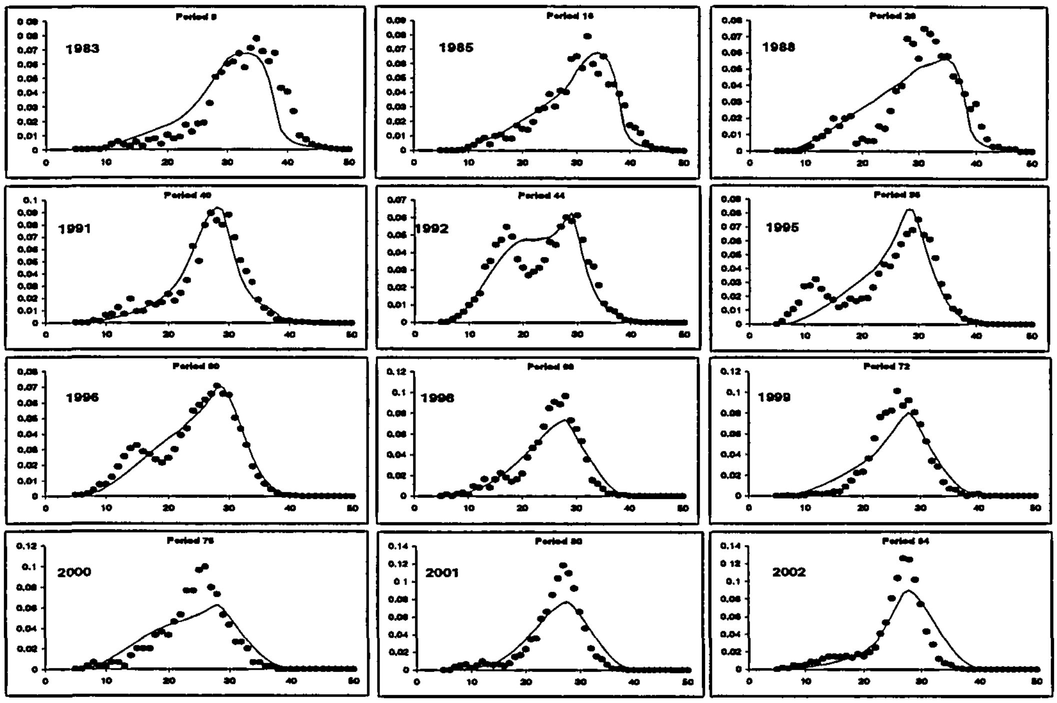


Fits to commercial shed sampling data

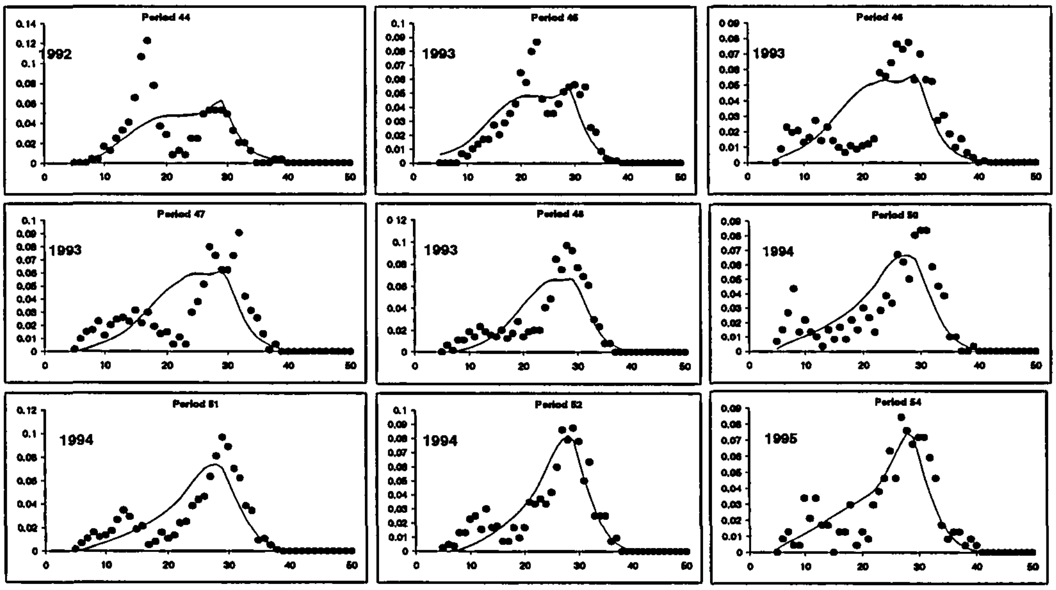


Appendix 9: **Lognormal error model fits to the survey and commercial length frequency data under high weighting toward the survey length frequency data (*weighting ratios* biomass : commercial LF : survey LF 1:5:20).**

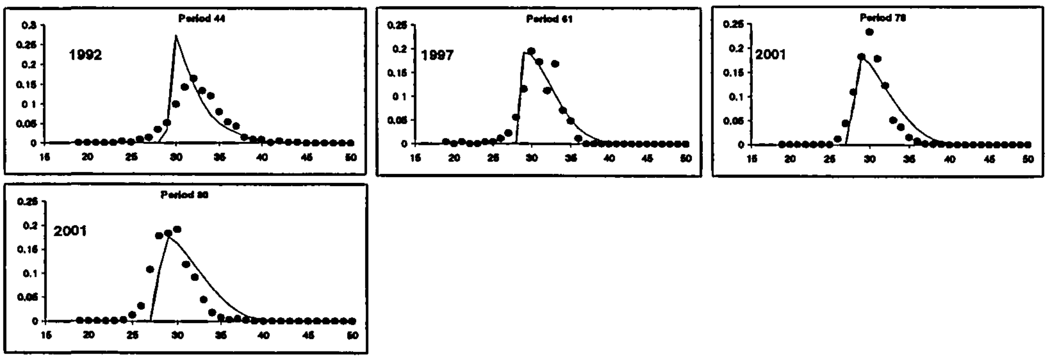
Fits to Survey Length Frequency data 1:5:20



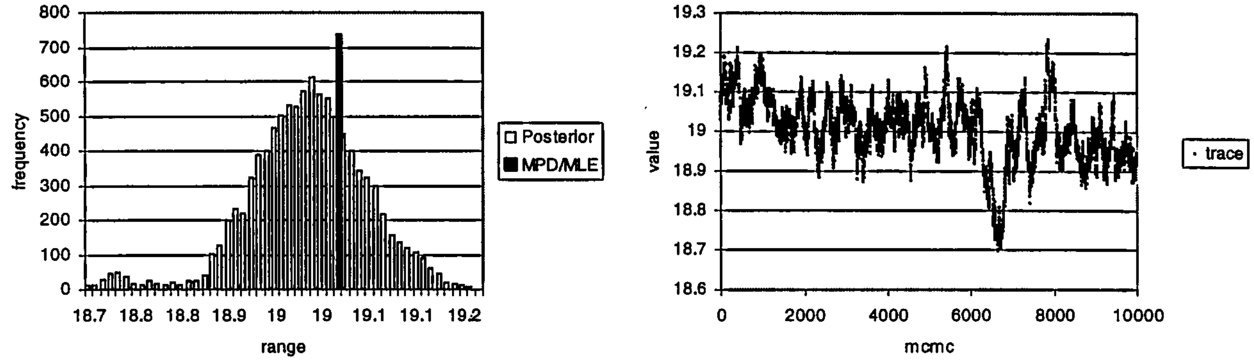
Fits to Seasonal sampling data



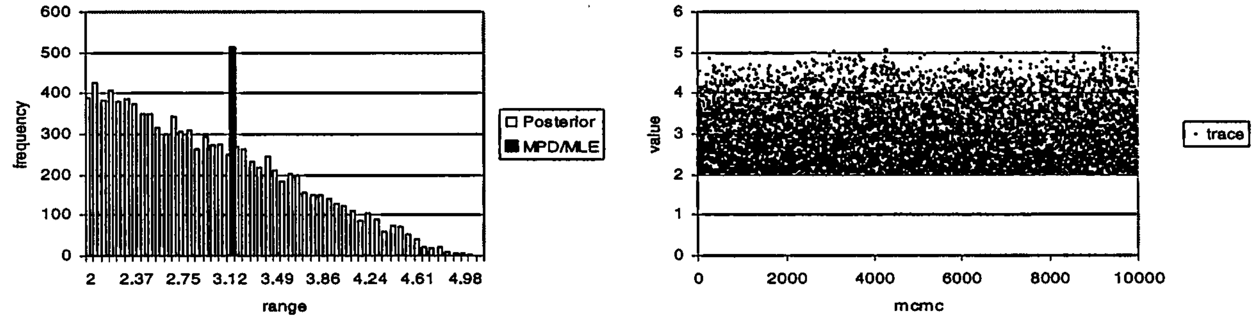
Fits to commercial shed sampling data



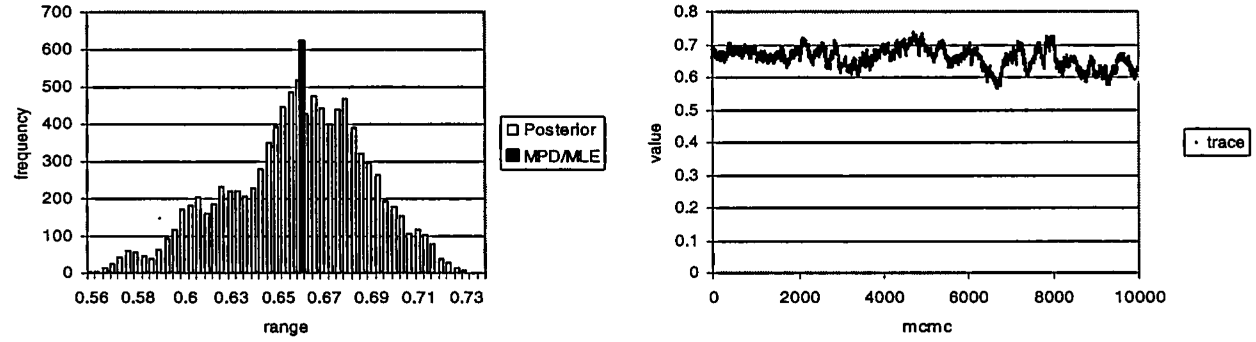
Appendix 10: Lognormal model posterior distributions and traces (chain length 10 000 sampled from 1 000 000 MCMC steps).



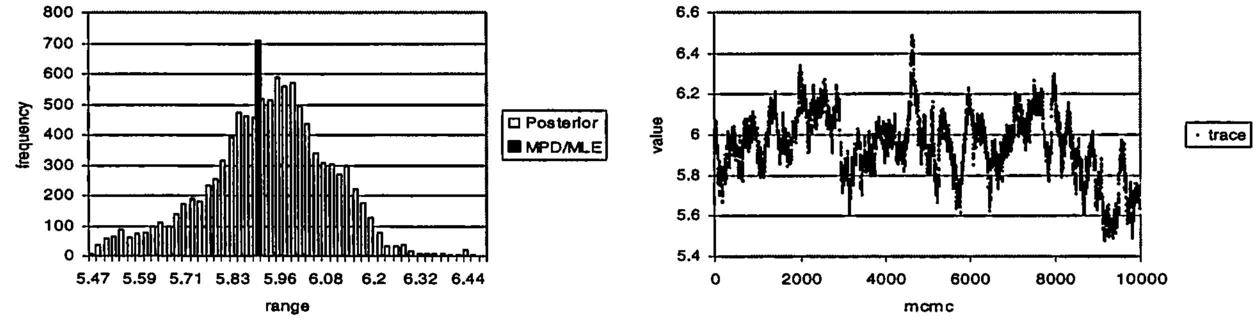
a. Log of recruitment *ln(R0)*



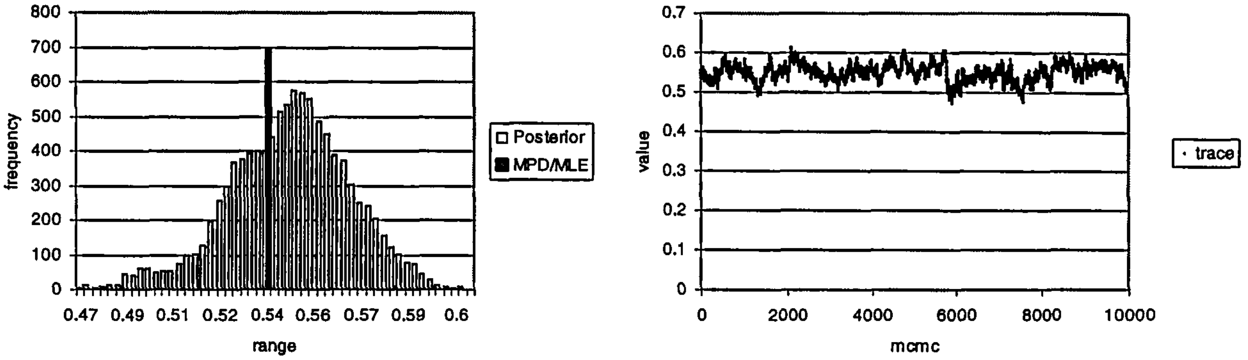
b. Length at first recruitment l^R



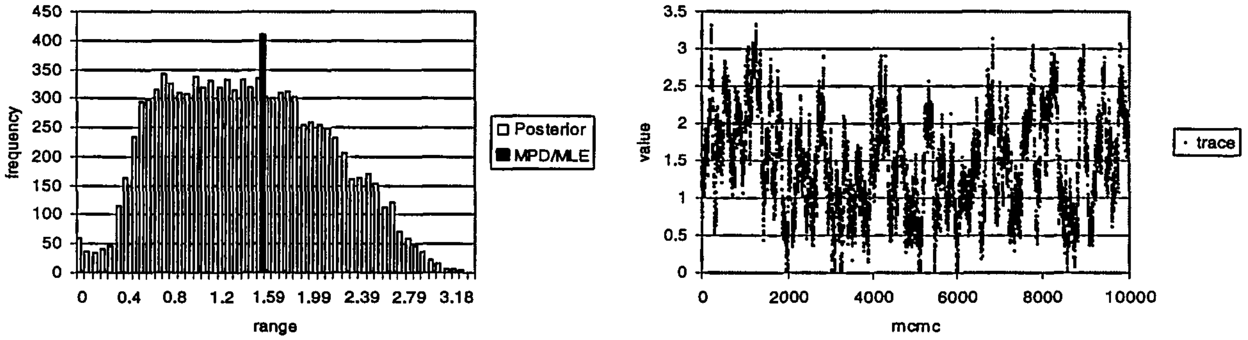
c. Natural Mortality *M*



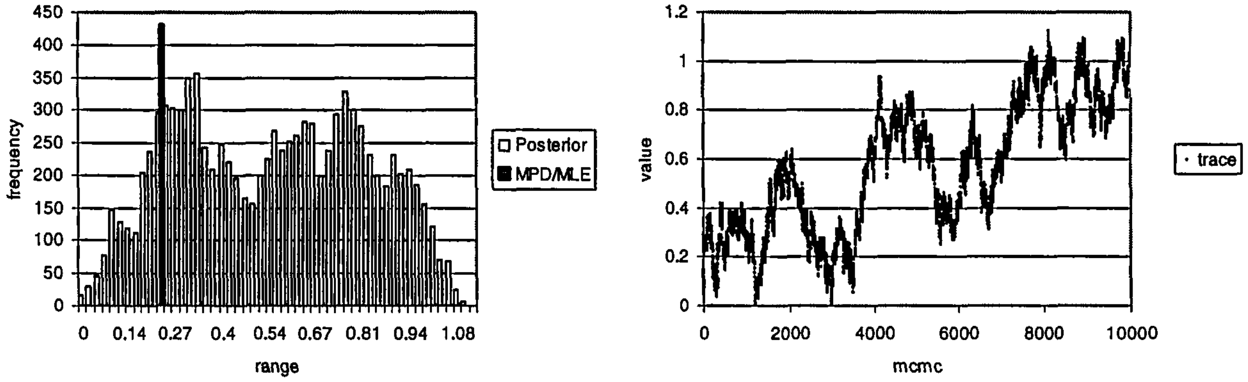
d. g 10



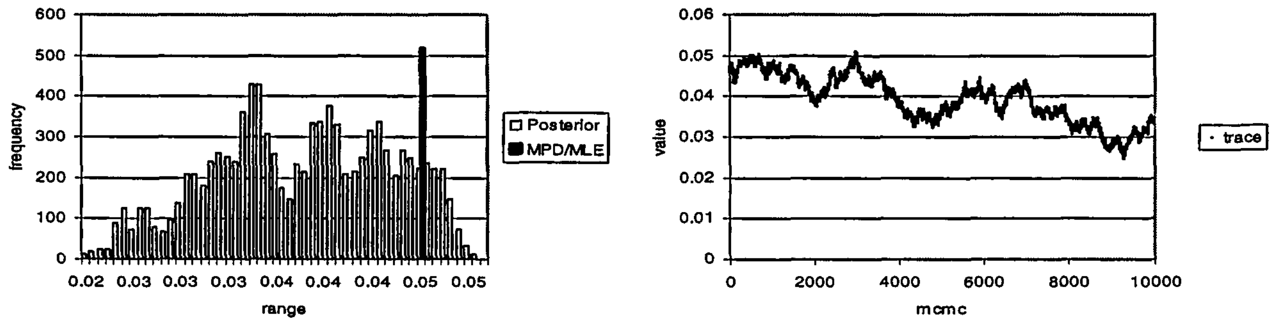
e. g 30



f. shape parameter for the left hand limb of research survey selectivity v^{1r}



g. shape parameter for the left hand limb commercial survey selectivity v^{1s}



h. Annual difference in commercial selectivity mean ϕ^{diff}