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A mixture model approach to derive within-season, cohort-specific CPUE for arrow squid (*Nototodarus sloanii*)

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TABLE OF CONTENTS

PLAIN LANGUAGE SUMMARY

Arrow squid represent an important fishery resource, but their fast growth and relatively short lifespan make stocks difficult to assess and manage. This study focused on developing a novel method to distinguish different squid cohorts based on size and catch-per-unit-effort (CPUE), across different areas and over time. Using information from trawl fisheries on the Stewart-Snares-shelf and at Auckland Islands, the assessment assumed two dominant cohorts in each area, occurring in autumn (early season) and spring (late season). This assumption was used to develop a mixture model (i.e., a probabilistic model) to determine the proportional contribution of each cohort to a given fishing event, which, in turn, allowed simultaneous estimation of CPUE for each cohort. The model indicated consistent depletion (i.e., decreases in squid over time) for the late-season cohort, but there was no consistent trend for the early-season cohort. This proof of concept highlights the potential to further develop this approach for providing stock assessments for squid in these trawl fisheries.

EXECUTIVE SUMMARY

Neubauer, P.[1]**; Large, K.**[1]**; Tornquist, M.G.**[1]**; Middleton, D.A.J.**[2]**; Tremblay-Boyer, L.**[1] **(2026). A mixture model approach to derive within-season, cohort-specific CPUE for arrow squid (*Nototodarus sloanii*).**

***New Zealand Fisheries Assessment Report 2026/05*. 29 p.**

This analysis modelled observed length distributions of the New Zealand arrow squid (*Nototodarus sloanii*) fishery in an attempt to separate length cohorts in space and time, with a view to deriving catch-per-unit-effort (CPUE) indices that may track the successive depletion of these cohorts by the trawl fishery in SQU 1T (Stewart-Snares shelf) and SQU 6T (Auckland Islands).

To gain an understanding of length distributions over spatial and temporal scales, a range of plots were inspected for patterns in length (presented in a companion report, characterising the fishery). These analyses suggested coherent, but somewhat temporally separated, length dynamics at Auckland Islands and the Stewart-Snares shelf. The patterns further suggested the presence of two dominant cohorts in most years in each area; however, this pattern was not evident in all years, and, in some years, more than two length cohorts may be present.

Based on the descriptive analyses, we constructed a mixture model to predict whether a fishing event was fishing on an early-season (e.g., autumn) or late-season (e.g., spring) cohort. The model was then used to estimate cohort-specific CPUE, providing a proof-of-concept for jointly estimating cohort contributions to fishing events based on fishery length frequencies, and estimating within-season CPUE from these cohorts. The model was applied to a subset of years with considerable observer coverage, for which patterns in CPUE suggested consistent depletion of numbers in the late cohort, while also showing little consistency in trends for the early cohort in both areas (although the early cohort was only consistently fished at Auckland Islands for the years considered in the present model).

The development of an integrated model to capture fishing on separate cohorts, and the temporal evolution of CPUE within seasons, opens the possibility to apply within-season DeLury depletion type models for SQU 1T and SQU 6T. Although there are a number of improvements and extensions to the present model that would likely improve the accuracy of estimated trends by providing a closer match to data, the present model demonstrates the potential to develop such models, and to derive potentially valuable indices of cohort abundance.

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1. INTRODUCTION

The southern arrow squid (*Nototodarus sloanii*) is fished in New Zealand’s southern waters around Auckland Islands and the Stewart-Snares shelf. Although information on its fishery biology and ecology was comprehensively summarised by successive characterisation reports (Uozumi 1998, Hurst et al. 2012, Middleton et al. 2026), important questions about the species’ biology and ecology remain. Previous attempts to develop assessment and management approaches have been unsuccessful (McGregor & Large 2016), primarily owing to the complexity of the squid population structure and limited information on the parts of the life cycle when the squid is not being fished.

Although no recent squid ageing data are available, Uozumi (1998) provided ageing and growth information for this species. Both fishery and biological data were reviewed in detail by Hurst et al. (2012) and Middleton et al. (2026). The lifespan of *N. sloanii* is considered to be close to one year, with highly variable growth and cohort strength – here, cohort is loosely defined as squid hatching at a similar time of year. While there may be dominant spawning times, available information suggests year-round spawning and recruitment to juvenile and adult habitat (Uozumi 1998). Nevertheless, the location of spawning is largely unknown: observations of late-stage squid are rare in fisheries observer data (Middleton et al. 2026), suggesting that spawning occurs outside of fished areas or seasons.

The targeted fishery on the Stewart-Snares shelf and around Auckland Islands mainly occurs in late summer and autumn, often on what appears to be two relatively large length cohorts (i.e., squid of similar size, which may or may not be of similar age). The first cohort observed in the fishery is presumed to have spawned in autumn or winter of the previous season, whereas fishing later in the season may target a potentially strong spring cohort (Middleton et al. 2026); however, these patterns appear somewhat variable between years and areas. Given this lack of knowledge, no assessment and management strategy has been adopted for New Zealand’s squid fisheries to date.

Internationally, management of effort, coupled with in-season assessment of cohort size, with the objective of allowing sufficient escapement has been suggested as the most appropriate form of management for squid fisheries (Beddington et al. 1990, Arkhipkin et al. 2015, 2021). This suggestion was due to the short-lived nature of these species, and the delay in assessment and management cycles often used for longer-lived finfish and crustaceans Hurst et al. (2012). This context suggested that this kind of approach may be worth investigating for New Zealand’s target fisheries.

McGregor & Tingley (2016) and McGregor & Large (2016) attempted to apply DeLury-type depletion models, based on models successfully applied at Falkland Islands, to fisheries in SQU 1T and SQU 6T. Although the approach showed potential, these studies could not obtain consistent depletion estimates for these fisheries when applied to nearly two decades of in-season fisheries data. This finding suggests that additional complexities in New Zealand’s squid fisheries may preclude a straightforward application of these methods.

A key difficulty with the DeLury depletion approach is that cohorts are assumed to be independent and fished down over time. Although extensions to the model have been made to accommodate in-season recruitment events (Arkhipkin et al. 2021), defining the timing and presence of these events in “noisy” in-season fisheries data provides a challenge. McGregor & Large (2016) used in-season increases in catch-per-unit-effort (CPUE) to suggest the presence of additional recruitment to the fishery. Nevertheless, there are a number of alternative hypotheses that can lead to increases in CPUE within season (e.g., vessel movement to new areas with unfished squid aggregations from the same cohort). Determining which cohort a given fishing event targeted, therefore, remains unknown, with likely significant implications for the applicability of DeLury depletion modelling, and the application of in-season effort controls.

We used visualisations of lengths collected by observers to understand where and when cohorts may appear in space and time (making the assumption that squid of similar length were spawned at a similar time). Based on these empirical data visualisations, we constructed a mixture model to predict whether a fishing event was fishing on an early (e.g., autumn) or late (e.g., spring) cohort. The model was then used to estimate cohort-specific CPUE. The model provides a proof-of-concept for jointly estimating cohort contributions to fishing events, based on fishery length frequencies, and estimating within-season CPUE from these cohorts. We applied this model to a subset of years with considerable observer coverage; we also suggest ways in which the application of the model could be further investigated, and discuss potential improvements that could allow within-season depletion methods to be applied.

The development of the models was part of project SQU2020-1b, with the overall objective to "develop and test a management strategy for arrow squid in SQU 1T and SQU 6T based on historical data".

2. METHODS

2.1 Data

Squid are caught predominantly during the late summer and autumn months (February to April; Figure 1), by a target fleet which has orders of magnitude higher catch rates of squid than other fisheries within which squid are regularly caught (Figure 2). The fishery also predominantly operates in particular areas and depths (Middleton et al. 2026), distinguishing it from other bottom- and midwater-trawl fisheries in the region. For this reason, the present project focused on targeted squid fishing effort, and did not consider other fisheries which may not fish the same component of the squid stock.

For observer data for all bottom and midwater trawl in SQU 1T and SQU 6T were extracted, for any trips between 2008 and 2020 that caught SQU (arrow squid, *N. sloanii* and *N. gouldi*), NOS (New Zealand southern arrow squid, *N. sloanii*), or NOG (New Zealand northern arrow squid, *N. gouldi*). For mixture analyses, only statistical areas covering the Stewart-Snares shelf (Statistical Areas 024, 025, 026, 027, 028, 029, 030, 504) and Auckland Islands (Statistical Area 602) were retained, which represented fisheries targeting New Zealand southern arrow squid (NOS). Areas of potential species mixtures were not retained for this purpose (i.e., Chatham Rise). Nearly all observed effort (98%) in the over-all area was from squid target trawls, using bottom (73%) and midwater trawls (23%). All remaining observed effort with squid catches (i.e., barracouta, hoki target effort) accounted for less than 1% of observed effort. Data for analyses were limited to years post 2008, when observer data covered a greater proportion of the fishery than in earlier years (Figure 3).

Trends in catch per tow in the observed portion of the targeted squid fishery (Figure 4) largely corresponded to seasonal CPUE trends in the wider fishery (see Figure 1) for squid, owing to the high proportion of squid catch occurring in the target fishery.

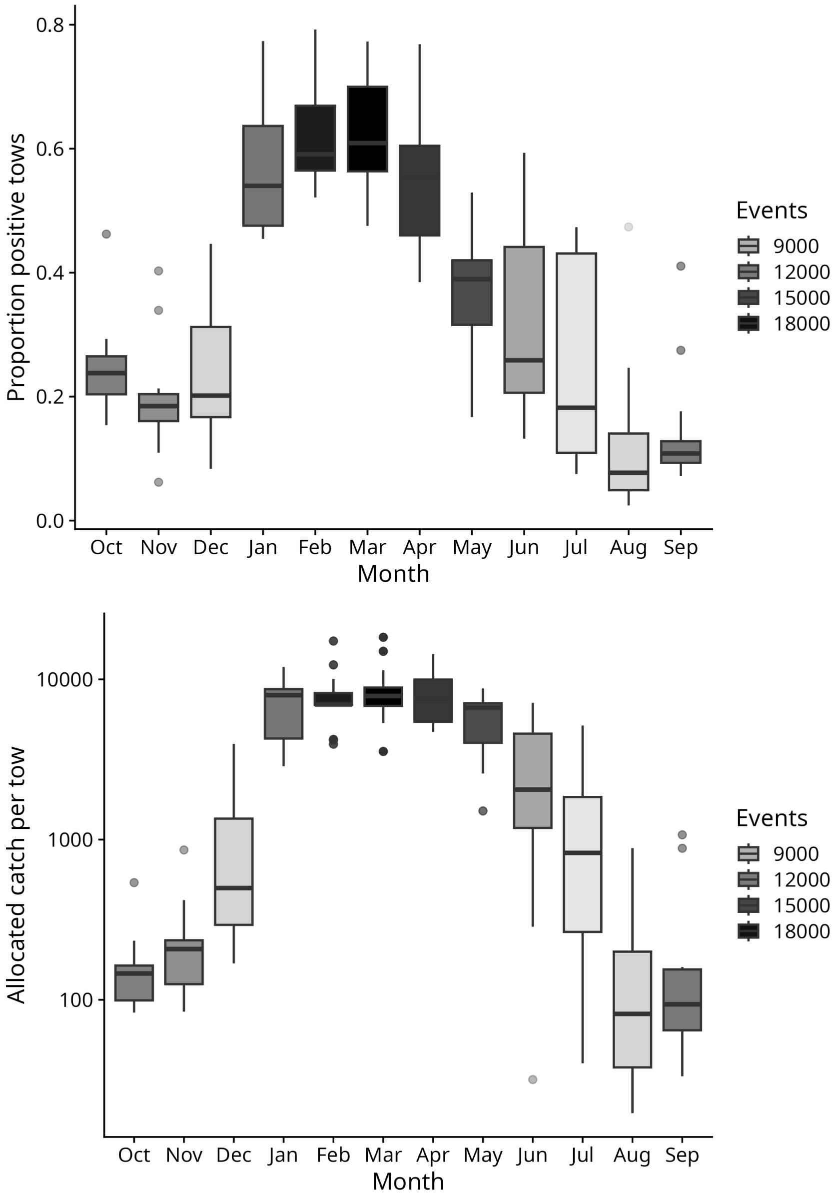

Figure 1: Monthly distribution of the proportion of events reporting squid (NOS, SQU) catch, and of allocated catch (t) per tow. Data are for tows reporting squid catch in Fisheries Management Areas SQU 1T and SQU 6T, for the period between 2008 and 2020.

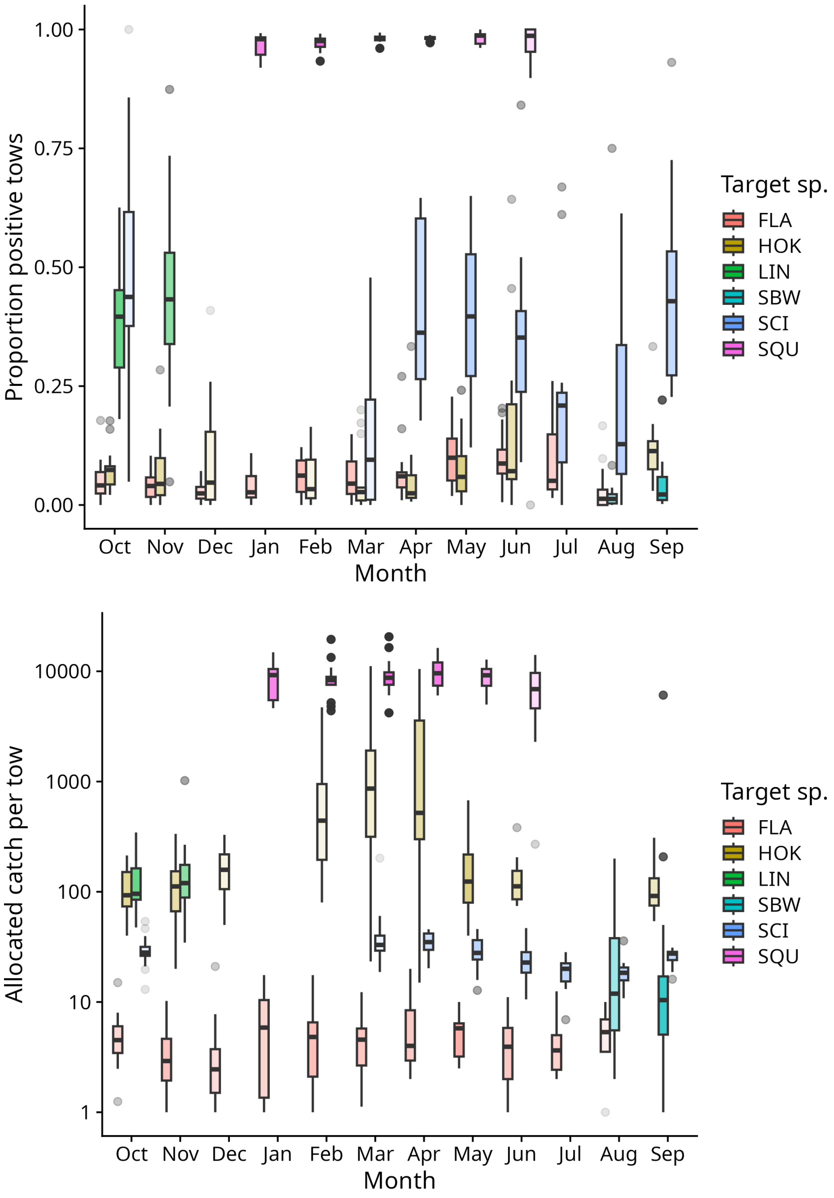

Figure 2: Monthly distribution of the proportion of fishing events reporting squid (NOS, SQU) catch, and of allocated catch (t) per tow. Data are for tows with reported squid catch in Fisheries Management Areas SQU 1T and SQU 6T, by target species, for the period between 2008 and 2020. Fisheries targets were: FLA, flatfish; HOK, hoki; LIN, ling; SBW, southern blue whiting; SCI, scampi; SQU, squid. Transparency indicates the number of events within each target fishery.

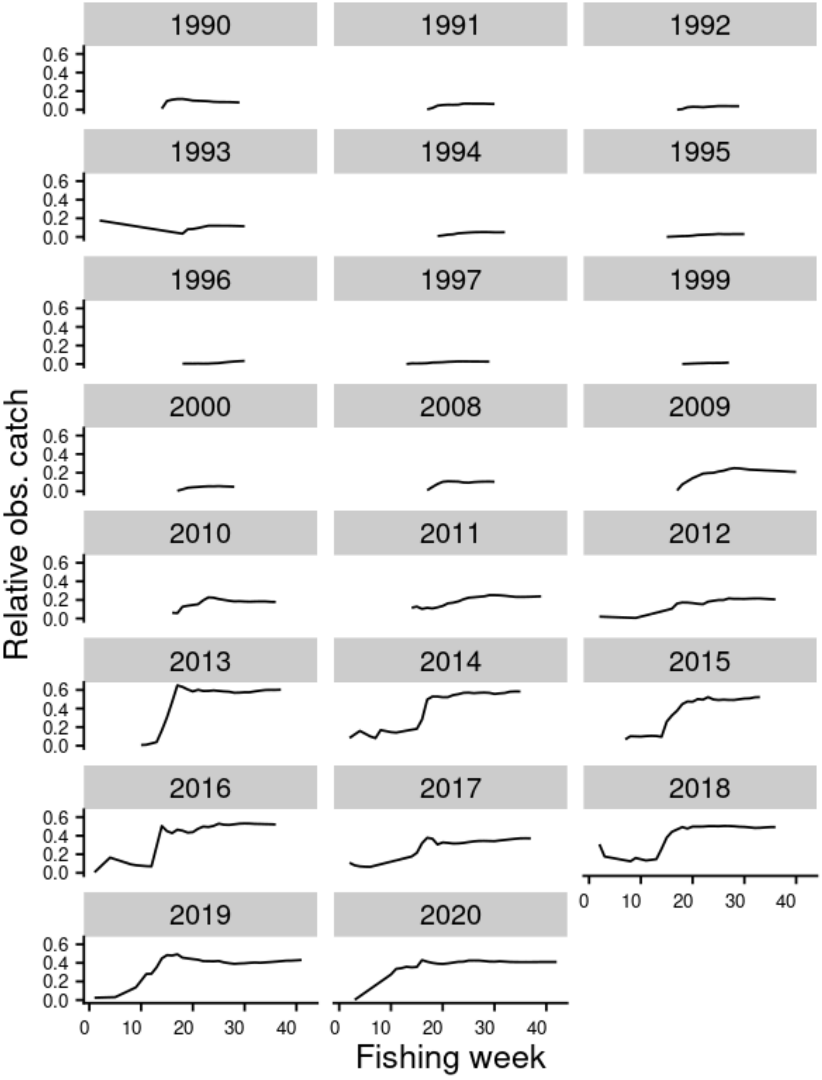

Figure 3: Observed proportion of squid target catch, by year and fishing week (with week 1 beginning on 1 October each year).

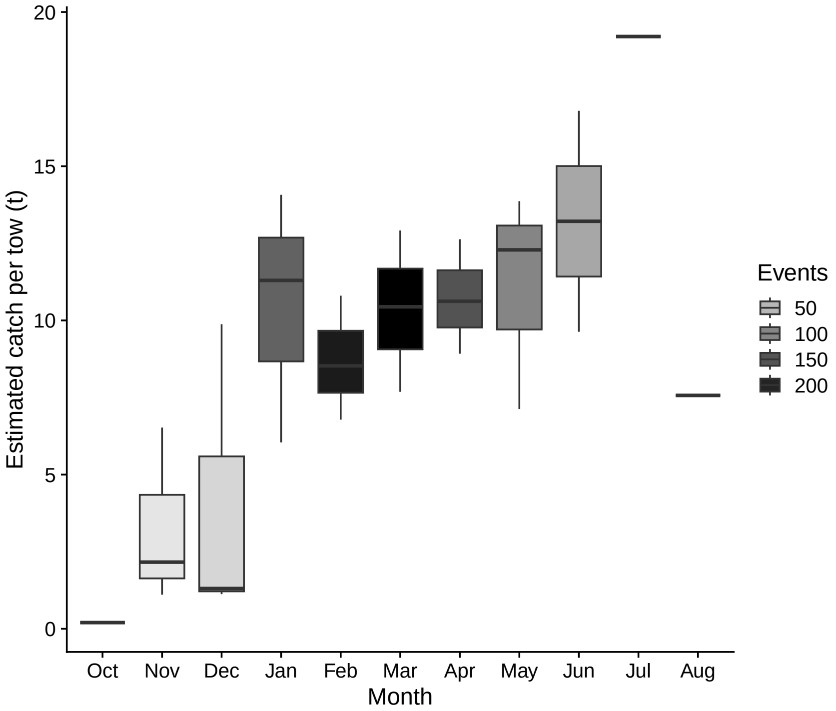

Figure 4: Estimated catch of squid (NOS, SQU) for observed fishing events targeting squid in SQU 1T and SQU 6T by month for the period from 2008 to 2020.

2.2 Mixture model of in-season cohort proportions

Based on empirical data visualisations (Figures 5, 6), we observed that the observed lengths were spatially variable, with length progressions of dominant cohorts evident through time in most areas and years. In addition, it appeared as though in most years, a dominant cohort of large squid was fished early in the fishery (e.g., prior to about week 20), with a second cohort being fished later in the season. Based on these observations, we hypothesised that: i) there are usually two dominant cohorts in all areas; but ii) fishing of these cohorts is not synchronised between areas, and may be spatially separated within areas at relatively small spatial scales. Fishing events in a given area (e.g., Auckland Islands) at a given time may, therefore, target either of these cohorts, or a mix of both cohorts.

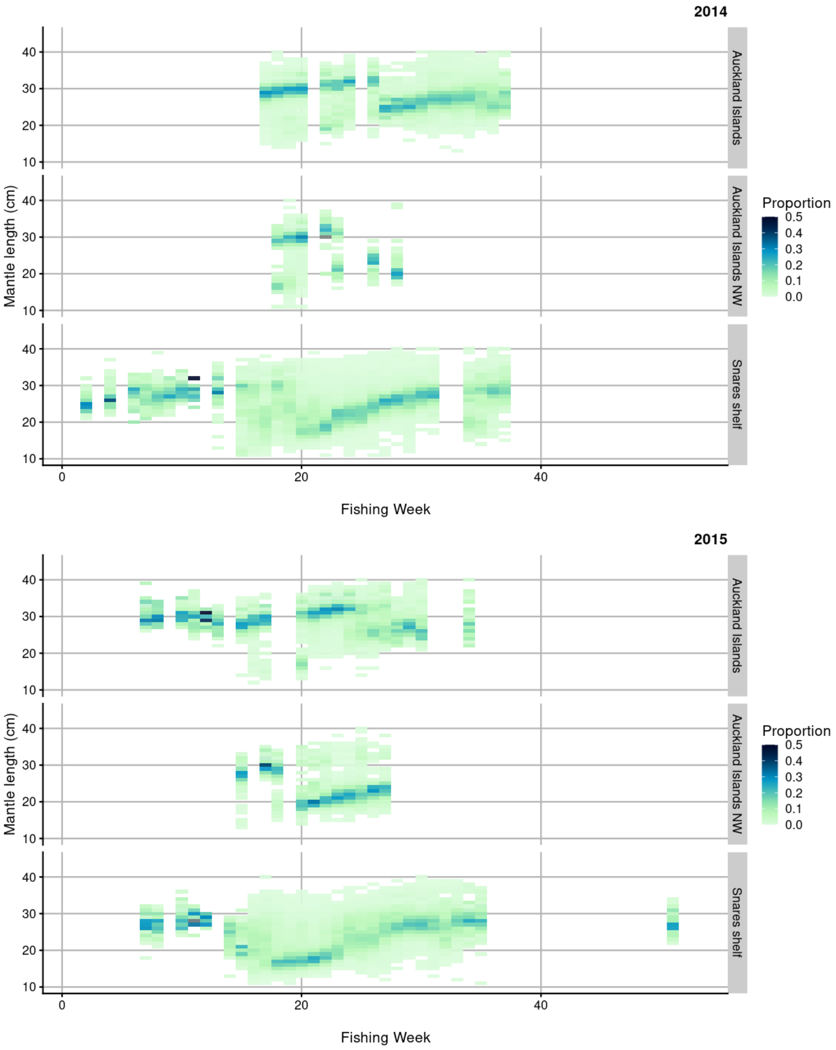

Figure 5: Aggregate length proportion distributions of squid for the 2014 and 2015 fishing years in each of three fishing areas. Colours indicate the proportion of squid at each length by fishing week (with week 1 beginning on 1 October each year). The Auckland Island NW area corresponds to the northwestern exposure of the Auckland Islands, compared with the more regularly fished south-eastern side of the islands. Areas are shown separately to illustrate the spatial variability in catch-at-length.

The above assumptions led to a mixture model over catch-at-length within a fishing event, with mixture proportions given by the proportional contribution of each cohort to the catch in the fishing event. Provided that observer catch sampling and measurements are unbiased, then observer length frequencies should be sufficient to estimate the mixing proportions, as long as the length distributions or catch rates of the cohorts are sufficiently separate for all locations and times.

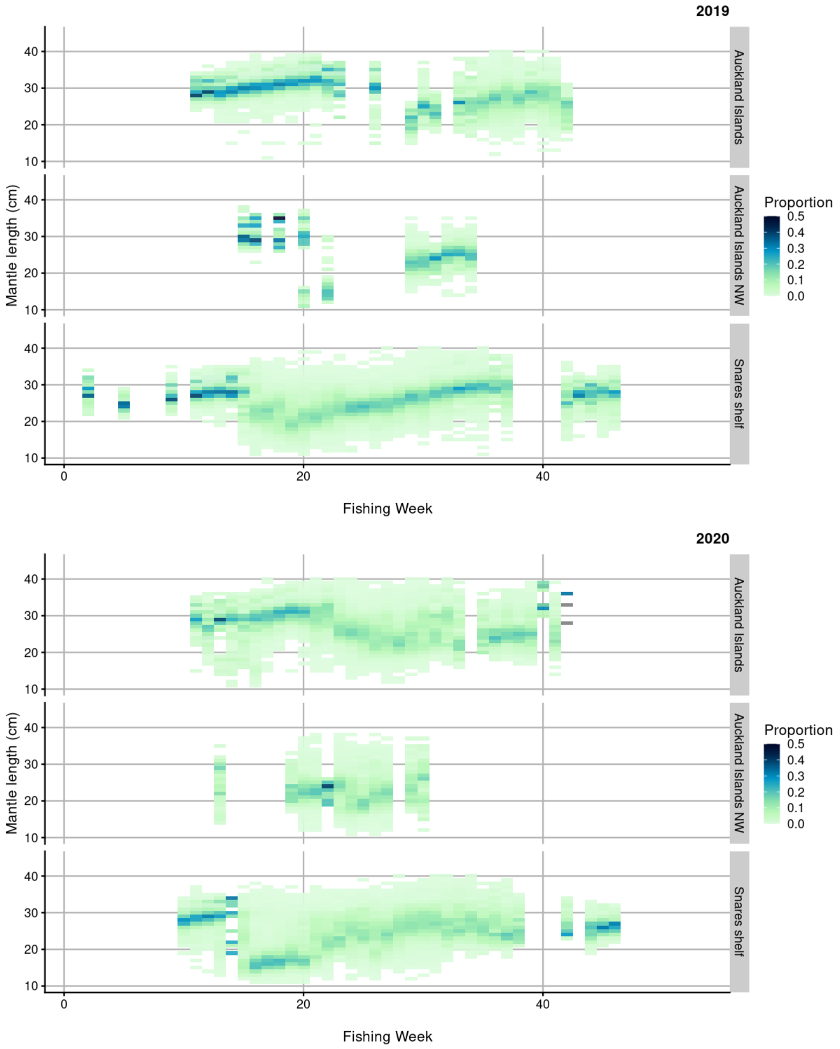

Figure 6: Aggregate length proportion distributions of squid for the 2019 and 2020 fishing years in each of three fishing areas. Colours indicate the proportion of squid at each length by fishing week (with week 1 beginning on 1 October each year). The Auckland Island NW area corresponds to the northwestern exposure of the Auckland Islands, compared with the more regularly fished south-eastern side of the islands. Areas are shown separately to illustrate the spatial variability in catch-at-length.

To be temporally relevant (i.e., to be able to be applied in-season, for a given fishing event), the model needs to account for in-season growth, which affects the expected length frequency for each cohort in space for a given week of the fishing season. We, therefore, modelled the expected length frequencies as arising from a von Bertalanffy growth model.

The mixture model aims to simultaneously estimate the parameters of the von Bertalanffy growth equation for each of the two assumed cohorts in a year, while also estimating the proportions of each cohort that contribute to fishing event. In addition, the model directly relates catch-at-length to CPUE from each cohort, standardised for common variables such as fishing duration and vessel. This modelling is accomplished by fitting the CPUE-at-length in observed fishing events with a mixture model consisting of two mixture components that correspond to the expected CPUE-at-length, derived by combining the von Bertalanffy models for each of the two cohorts for a given week and year, with the expected CPUE (relative abundance) of that cohort. The mixing proportions (i.e., the probability, for each cohort, that an individual with length l in a tow from a given event i came from cohort c) are given by the mixture probabilities θ.

Conceptually, the model is most straightforwardly presented in terms of latent discrete parameters $\hat{c}_i$, which represent the estimated cohort label for event i, given mixture proportions θ, which will vary through time at each of the different areas (i.e., mixture proportions will be skewed towards early cohorts early in the season, and late cohorts later in the season). This relationship can be represented as:

$$\begin{aligned} \hat{c}_i &\sim categorical(\theta_i), \\ \theta_i &= logit^{-1}(\bar{\theta}_2 + \beta_w week_i + \beta_a area_i), \end{aligned} \tag{1}$$

where $\sim$ means “is distributed as”, and categorical is a categorical distribution with probabilities θ. This model is a logistic regression at a lower level of a hierarchical model, and β_w estimates how quickly fishing transitions from the first cohort (the “early” cohort) to the second cohort (the “late” cohort) over the course of a season across fishing weeks w, and β_a estimates an offset to the timing of transition between cohorts depending on the area. Conditional on the estimated cohort label for event i, $\hat{c}_i$, the mean squid length of observed fishing event i is then modelled as coming from a von Bertalanffy growth function:

$$\begin{aligned} f(l_i) &= t(l_i|\eta_{[\hat{c}_i]}, \zeta, v) \\ \eta_{[\hat{c}_i]} &= \Lambda_{[\hat{c}_i]}(1 - exp(-k(w_i - \Upsilon_{[\hat{c}_i]}))), \end{aligned} \tag{2}$$

where $\eta_{[\hat{c}_i]}$ is the expected length in week w_i, Λ is the asymptotic size (normally denoted L_∞, k is the growth coefficient, and Υ_c is the cohort-specific week when size is zero (normally denoted t_0). Cohort lengths in a given week are assumed to be distributed according to a student-t distribution with variance ζ^2 and v degrees of freedom, both of which are estimated in the model. This basic model could, in the future, be extended by allowing the timing and growth coefficients of the second cohort to vary across years (random effect on Υ_c and k); however, since the present model was initially fitted to a limited number of years, we assumed a single growth coefficient and growth varied only by timing for each of the two cohorts across years.

Catch-at-length for each event (tow) was assumed to be distributed according to a negative-binomial (NB) distribution with parameters σ_c (parametrised as $1/\varphi$, the negative binomial dispersion parameter). (Note that the negative binomial assumption here is a placeholder – its support over only integer values is not aligned with catch-at-length values. Nevertheless, given large catches, the rounding error here was expected to be small, and the current assumption avoided the need to model zeros separately in the model — many length classes will have zero catch for each tow). In addition, catch-at-length was derived by scaling observed length proportions to catch for each tow (note, that scaling proportions-at-length to overall catch in numbers-at-length does not consider sampling error in length proportions, which will lead to higher estimated dispersion for catch at length per tow (higher over-dispersion of the negative binomial sampling distribution). The model for catch $C_{i,l}$ at length l for event i is then:

$$\begin{aligned} C_{i,l} &\sim NB(\mu_{l_i,i,[\hat{c}_i]}, \sigma_{[\hat{c}_i]}), \\ log(\mu_{i,c_i,l_i}) &= \bar{\mu}_{c_i} + log(f(l_i)) + \beta_e X_i + \gamma_e Z_i + \gamma_c Z_{c_i}, \end{aligned} \tag{3}$$

where the $log(f(l_i))$ in the linear predictor serves to scale catch according to Equation 2. The remaining terms standardise catch with respect to effort variables in the same way as standard CPUE analyses are generally conducted using generalised linear (mixed) models. Standardisation of cohort-specific CPUE-at-length used event-level fixed (X_i: main fishing areas) and random effects (Z_i; vessel key, cubic smoothing splines for bottom depth, and fishing duration), and also cohort-specific effects (Z_{c_i} fishing year, week within year, and area) in a linear predictor linked to the mean of the negative binomial distribution.

The cohort- and area-specific week-within-year effect served as the within-season index of interest here, to assess if significant differences between areas and years exist in the timing each cohort's CPUE trends. Alternative indices could be formulated and compared, such as non-area specific indices for season-weeks within years, reflecting the alternative assumption that cohorts appear simultaneously in both areas.

The season-week index derived here only covered observed events (we suggest implications for non-observed effort in the Discussion section). Due to the slow run time of the model, the model was fitted to three years of data, for 2013–14, 2014–15, and 2018–19, to facilitate development. The model was implemented in *brms*, using 16 cores per chain, and run in Stan using 1500 draws from Markov chain Monte Carlo (MCMC; see the appendix for the full model code). Convergence was assessed visually, and by inspecting uni- and multivariate Rhat statistics (Vehtari et al. 2021), which are commonly used metrics to assess convergence based on multiple concurrent MCMC chains.

CPUE for depletion modelling needs to index number depletion, rather than biomass depletion. We applied a model using observer length-frequency data to estimate length-weight relationships, and used estimated relationships to predict the weight of individuals at each length. CPUE by cohort and length was derived by partitioning cohort CPUE (catch C_i per standardised tow) into length components according to parameters estimated in Equation 2. The length-weight model took the form (using R formula notation):

```
log(fish_weight) ~ 1 +
           main_areas +
           log(length) +
           (1|fishing_event_key) +
           (1|fyear).
```

The model, therefore, allowed for variation in length-weight relationships among areas and years to be accounted for in scaling CPUE in catch weight to CPUE in numbers.

3. RESULTS

3.1 Mixture model of cohort CPUE

The mixture model converged to a stable solution, as long as reasonable starting values were provided (Figure 7). Once convergence was achieved, parameter estimates generally had relatively low uncertainty. In particular, estimates of growth and asymptotic size were both lower than suggested by the prior (Table 1; priors are given in Appendix A), which was derived from growth data presented by Uozumi (1998).

Standardisation variables had relatively little effect; fishing depth and duration had negligible effects due to an absence of clear within-season trends in fishing depth and duration (Figures 8, 9). Vessel, often a proxy for gear configuration, also had a relatively low standard deviation, compared with variation in individual fishing events, and between year and within-season changes (Table 1).

The model produced patterns of relative abundance-at-length that resembled patterns evident empirically (Figures 10, 11); however modelled patterns were smoother and more constrained than the observed data owing to the plotting in terms of standard tows. The model reproduced the incremental growth for each cohort (Figure 12), and followed the transition in the fishery from the early cohort (Cohort 1) to the later cohort (Cohort 2; Figure 13). The combination of these processes produced the patterns that reflected patterns evident across most years in the fishery (see Figures 10, 11). Residual patterns (Figures 14, 15) suggested that the 2019 cohort may have appeared slightly earlier; the model

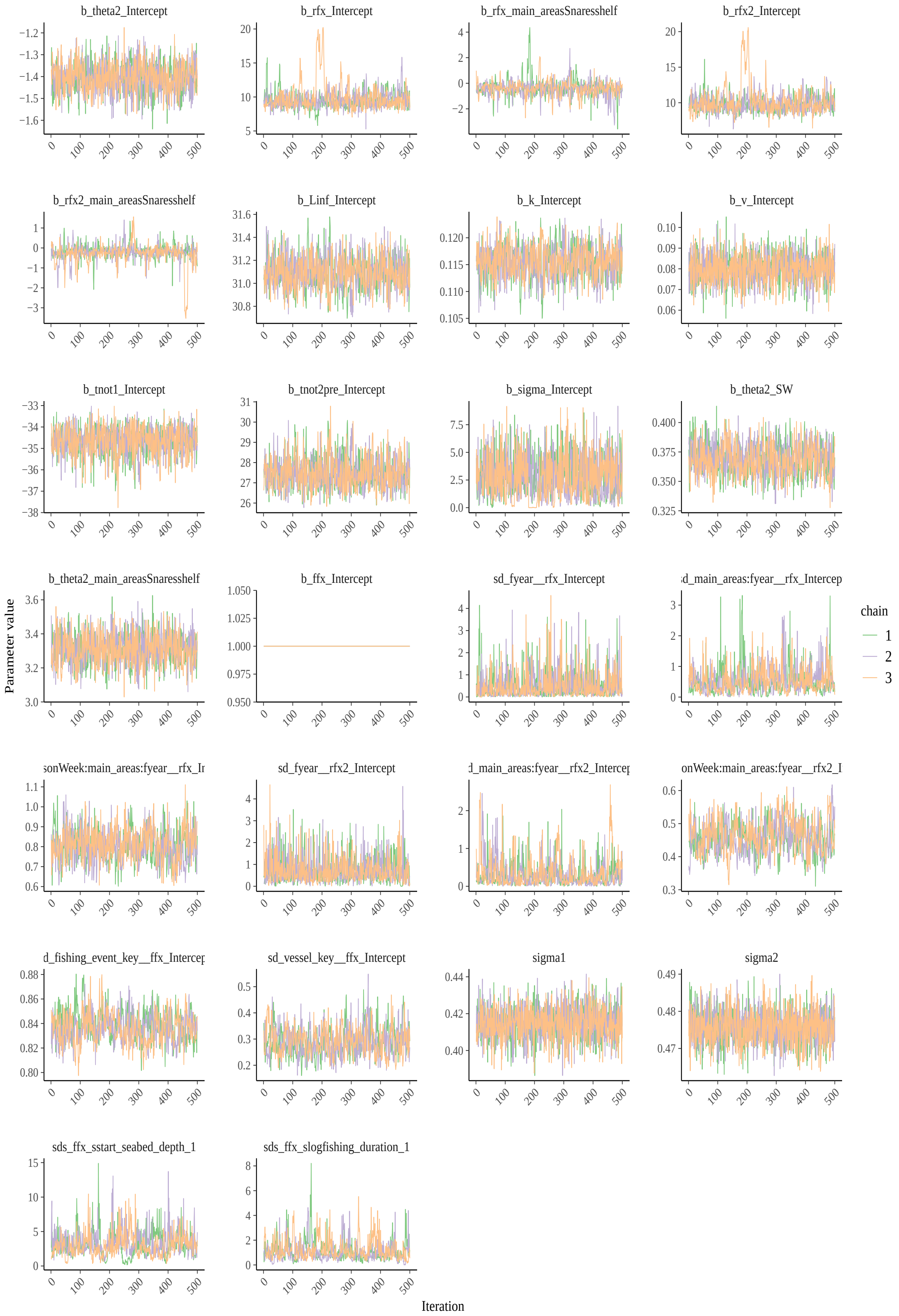

Figure 7: Markov chain Monte Carlo traces for the mixture model of squid catch-per-unit-at-length across cohorts.

Table 1: Posterior estimates (mean, median, and 95% confidence), convergence (Rhat; should be close to 1), and effective sample size (ESS) for model parameters in the mixture model for squid catch-per-unit-effort-(CPUE)-at-length.

Variable	Mean	Median	SD	CI	Rhat	ESS
θ_2 Intercept	-1.41	-1.41	0.07	$[-1.52; -1.30]$	1.00	865.93
Cohort 1 CPUE Intercept	9.72	9.45	1.61	$[7.98; 12.08]$	1.01	213.07
Cohort 1 Snaresshelf CPUE offset	-0.41	-0.40	0.62	$[-1.33; 0.43]$	1.00	496.30
Cohort 2 CPUE Intercept	10.02	9.74	1.57	$[8.26; 12.22]$	1.01	239.67
Cohort 2 Snaresshelf CPUE offset	-0.24	-0.21	0.43	$[-0.85; 0.29]$	1.01	346.53
Linf	31.09	31.09	0.14	$[30.87; 31.33]$	1.00	717.71
k	0.12	0.12	0.00	$[0.11; 0.12]$	1.00	716.64
nu	0.08	0.08	0.01	$[0.07; 0.09]$	1.00	1752.35
Y_1	-34.67	-34.61	0.67	$[-35.88; -33.71]$	1.00	903.57
Y_2	27.51	27.47	0.77	$[26.39; 28.90]$	1.00	792.78
ζ	2.79	2.54	1.88	$[0.25; 6.20]$	1.01	254.17
θ_2 Season-Week transition	0.37	0.37	0.01	$[0.35; 0.39]$	1.00	523.15
θ_2 Snaresshelf offset	3.31	3.31	0.09	$[3.16; 3.46]$	1.00	1002.29
sd fyear Cohort 1 CPUE	0.52	0.31	0.62	$[0.02; 1.72]$	1.01	572.17
sd main areas:fyear Cohort 1 CPUE	0.53	0.43	0.43	$[0.07; 1.33]$	1.02	273.85
sd SeasonWeek:main areas:fyear Cohort 1 CPUE	0.81	0.80	0.08	$[0.68; 0.94]$	1.00	276.87
sd fyear Cohort 2 CPUE	0.73	0.55	0.60	$[0.10; 1.98]$	1.00	872.78
sd main areas:fyear Cohort 2 CPUE	0.32	0.21	0.35	$[0.02; 1.07]$	1.01	309.38
sd SeasonWeek:main areas:fyear Cohort 2 CPUE	0.46	0.45	0.05	$[0.38; 0.54]$	1.01	223.40
sd fishing event key CPUE std	0.84	0.84	0.01	$[0.82; 0.86]$	1.02	193.35
sd vessel key CPUE std	0.29	0.29	0.05	$[0.21; 0.38]$	1.01	378.20
σ_1 (residual error Cohort 1)	0.41	0.42	0.01	$[0.40; 0.43]$	1.00	1515.94
σ_2 (residual error Cohort 2)	0.48	0.48	0.00	$[0.47; 0.48]$	1.00	1634.15
spline sd ffx start seabed depth	3.10	2.82	1.70	$[0.95; 6.10]$	1.01	159.58
spline sd ffx log(fishing duration)	1.14	0.93	0.81	$[0.31; 2.71]$	1.02	181.33

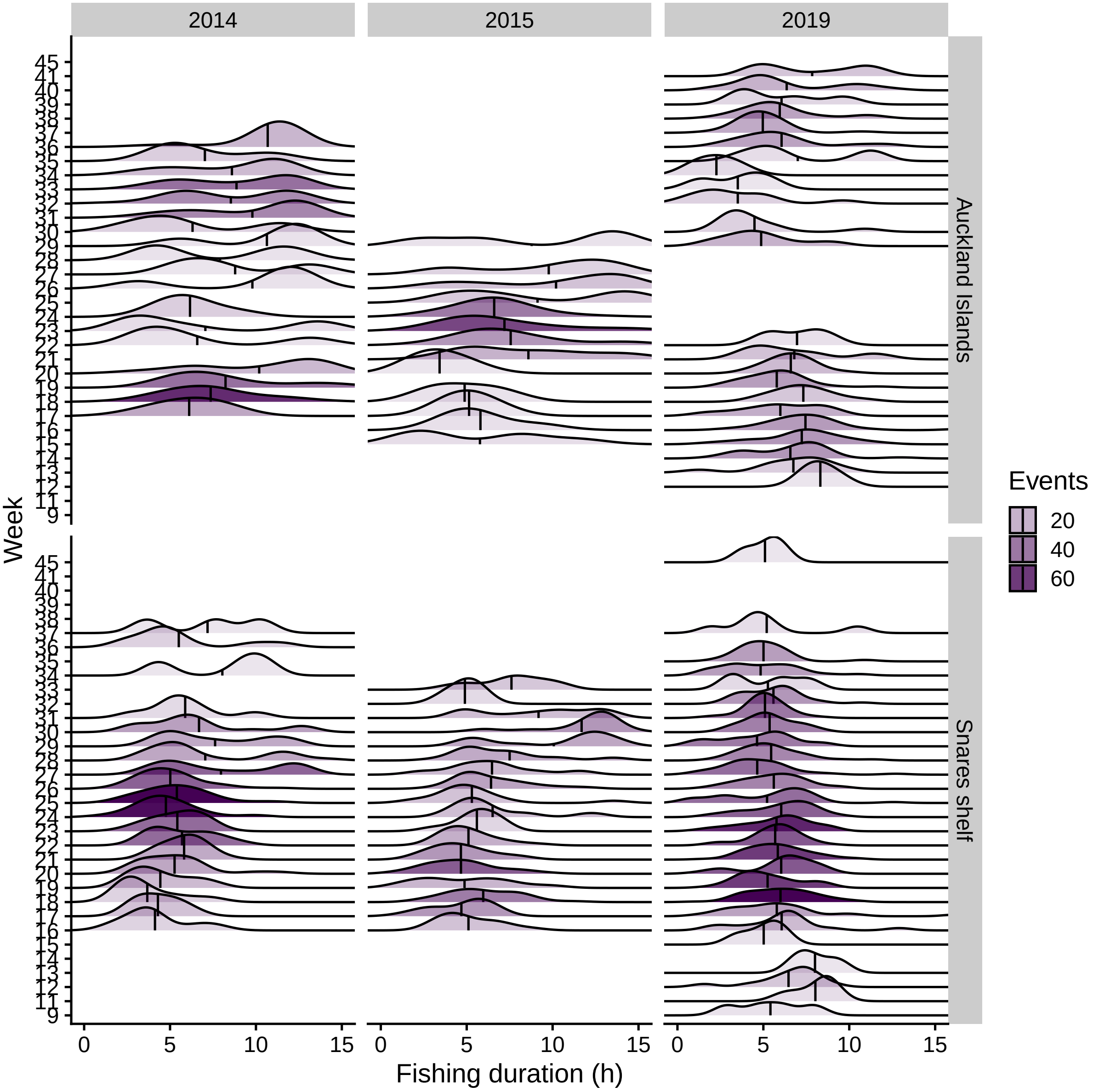

Figure 8: Reported fishing duration by week in each of the analysed fishing years. Week 1 begins on 1 October each year.

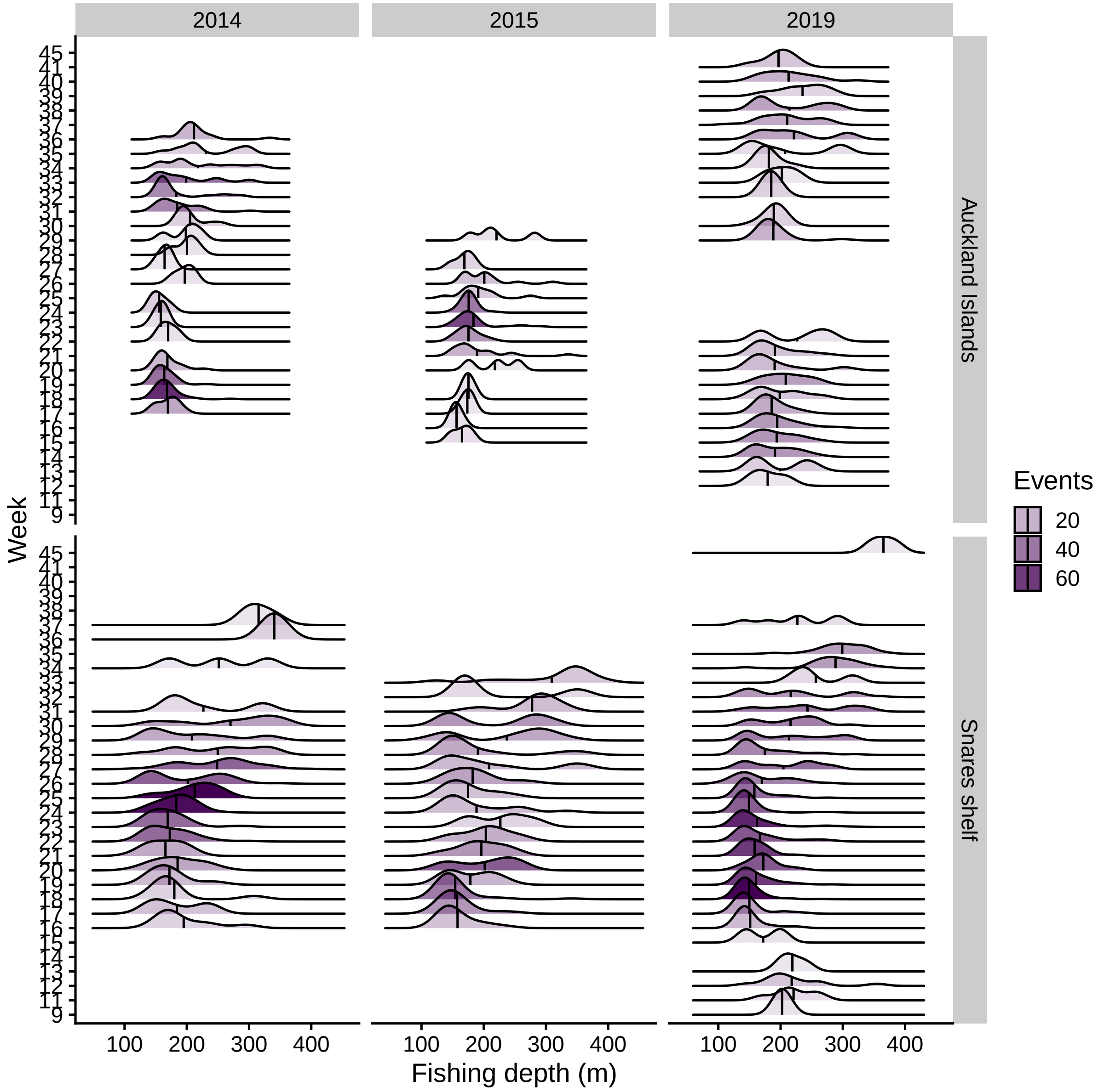

Figure 9: Reported fishing depth by week in each of the analysed fishing years. Week 1 begins on 1 October each year.

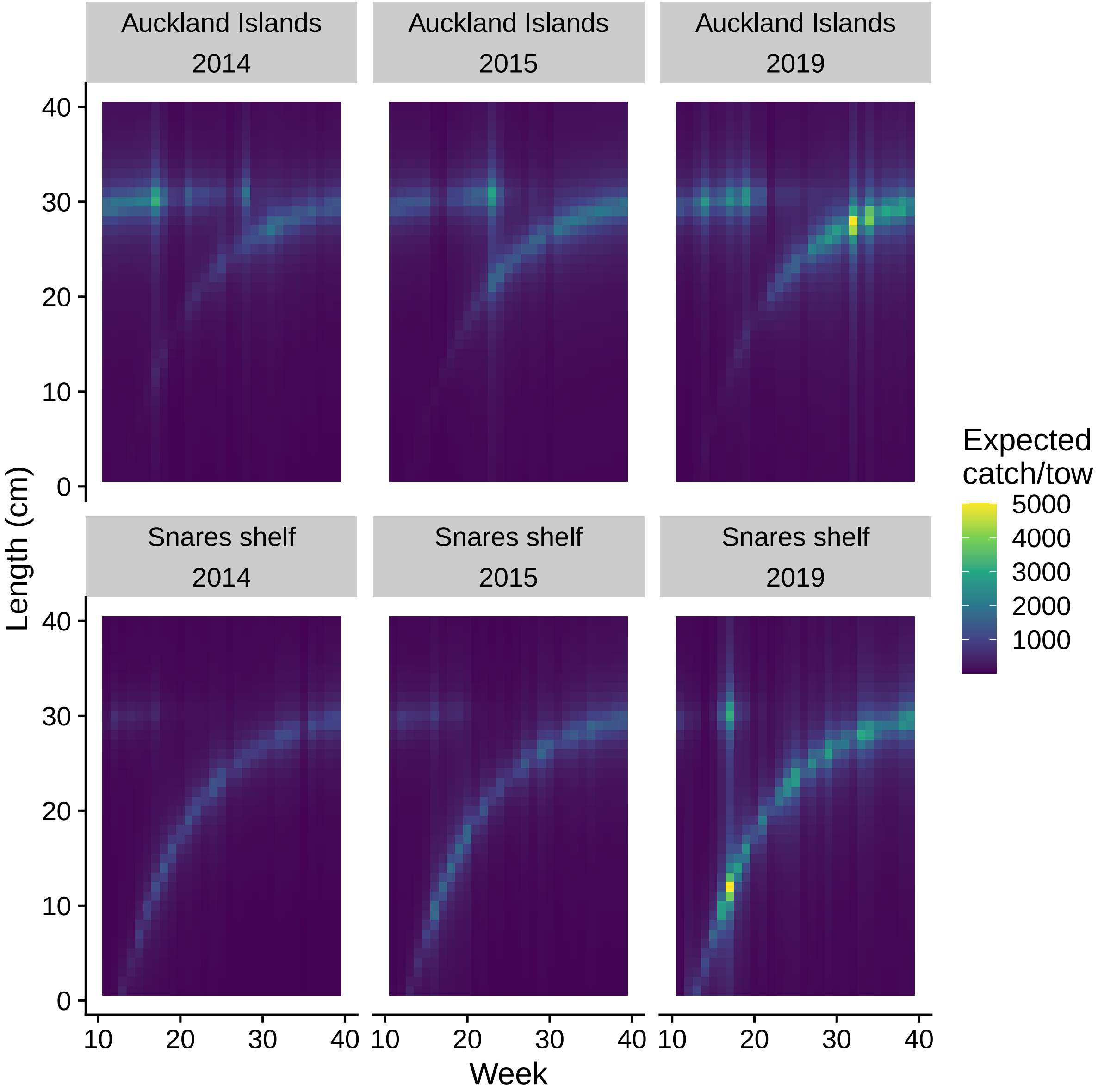

Figure 10: Expected catch (kg) per length and week in a standard tow in each of the two main areas and fishing years in the mixture model. Snares shelf, Stewart-Snares shelf.

could not currently reproduce this slight shift as it did not currently have parameters for timing of the second cohort that vary among years.

CPUE in weight showed a mixture of declining and increasing trends for the early cohort at both Auckland Islands and on the Stewart-Snares shelf (Figure 16). For the late cohort, however, a more clear systematic reduction in the in-season index was visible, particularly on the Stewart-Snares shelf. CPUE in numbers, derived from the length-weight relationship (Figures 17, 18), magnified these trends (Figure 19), with strong and consistent declining trends within season for the late cohort at the Stewart-Snares shelf. In addition, patterns at Auckland Islands were also more variable, with weak support for high early CPUE. Nevertheless, during the early weeks of the season, fishing was largely concentrated on large-sized squid from the early cohort at Auckland Islands. For this reason, there was considerable uncertainty about early trends.

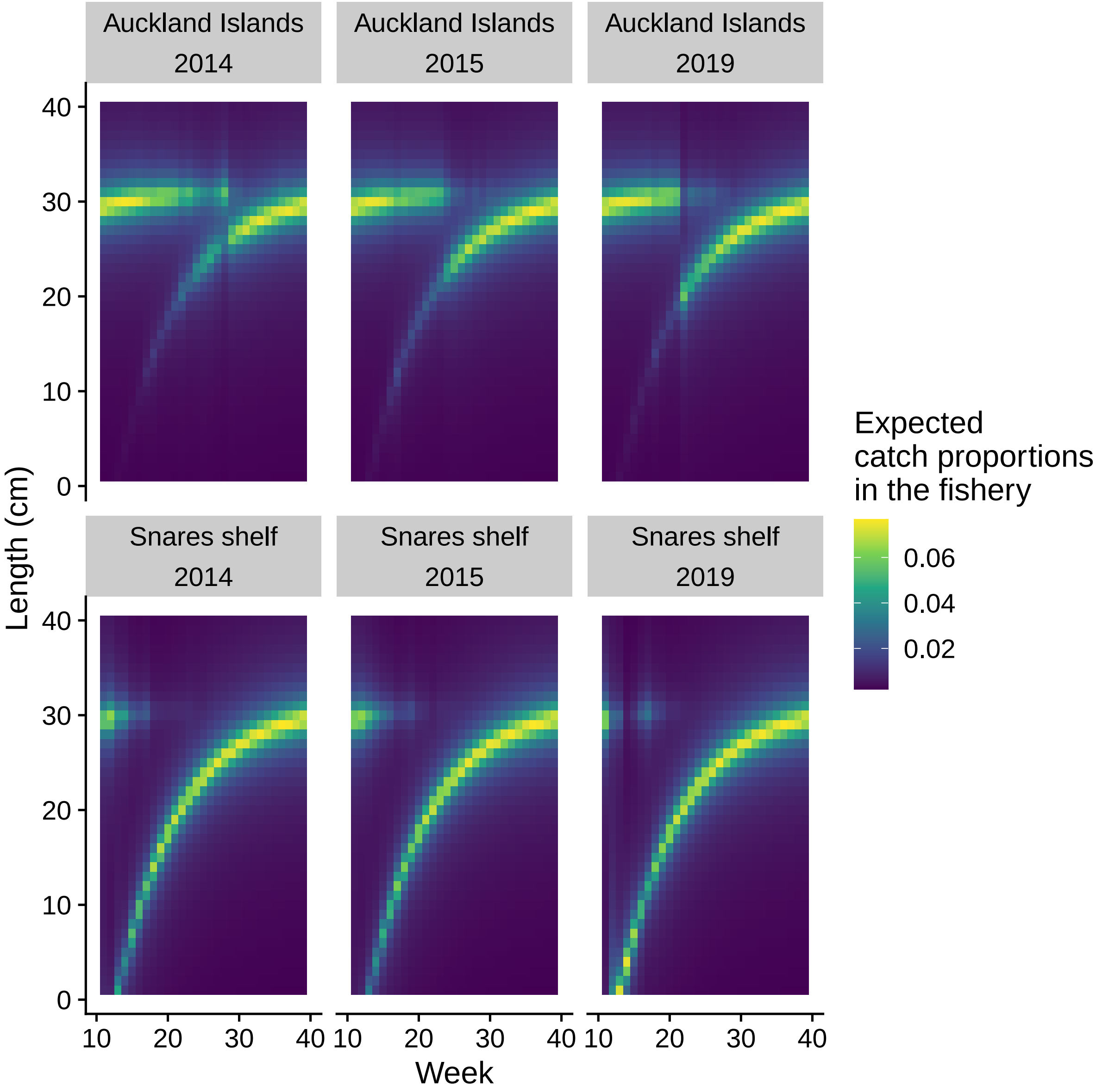

Figure 11: Expected proportions of catch weight at length by week in a standard tow in each of the two main areas and fishing years in the mixture model. Snares shelf, Stewart-Snares shelf.

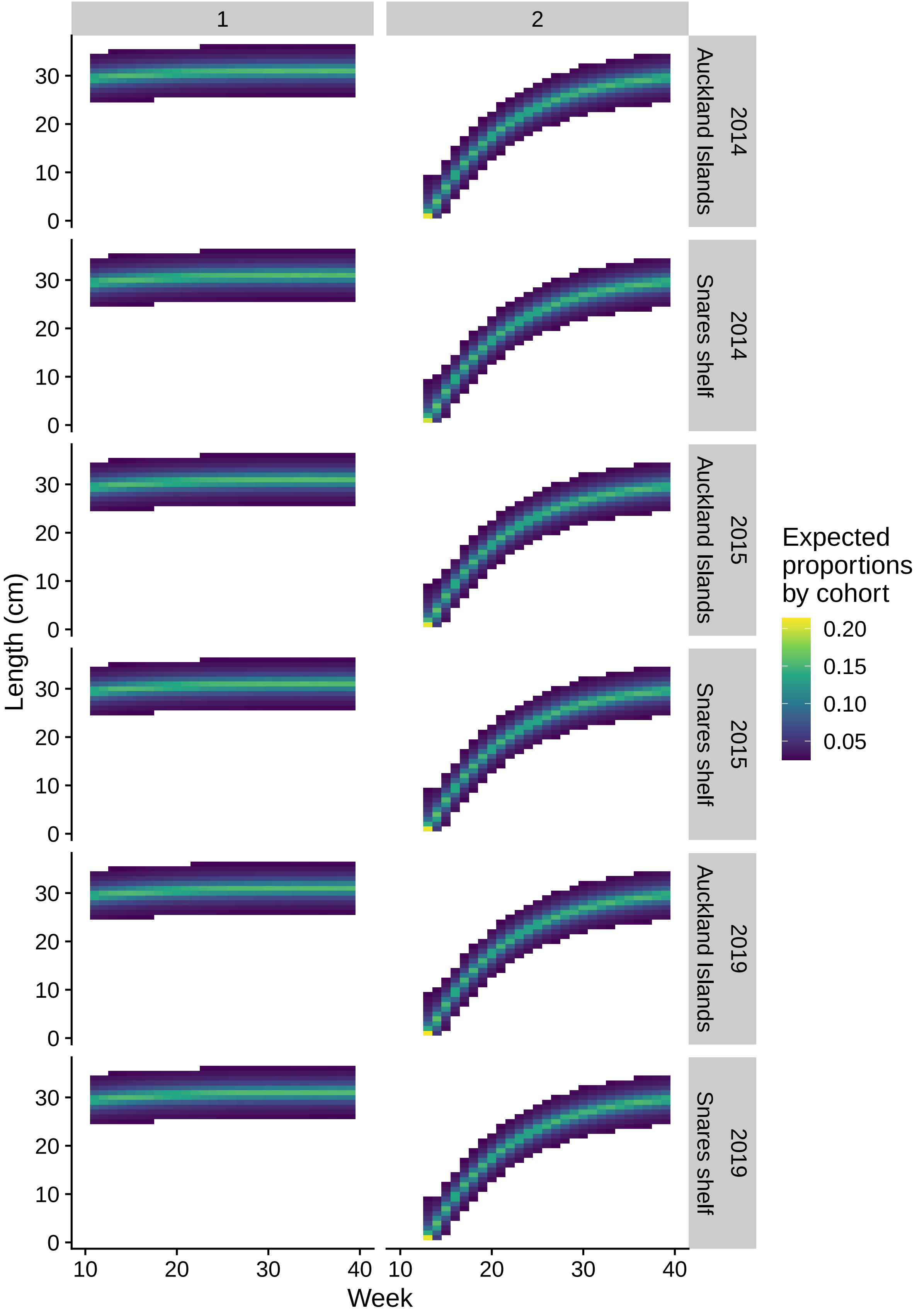

Figure 12: Expected number per length and week for each cohort in each of the two main areas and fishing years in the mixture model. Snares shelf, Stewart-Snares shelf.

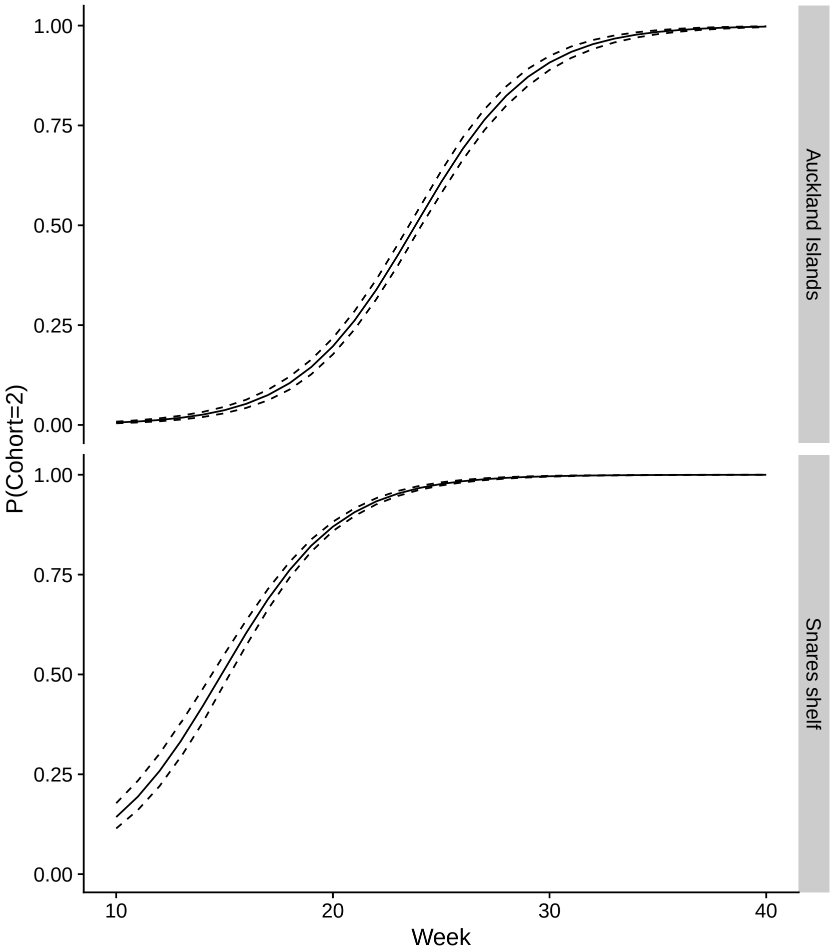

Figure 13: Estimated within-season transition between cohorts at each of the main areas. Snares shelf, Stewart-Snares shelf.

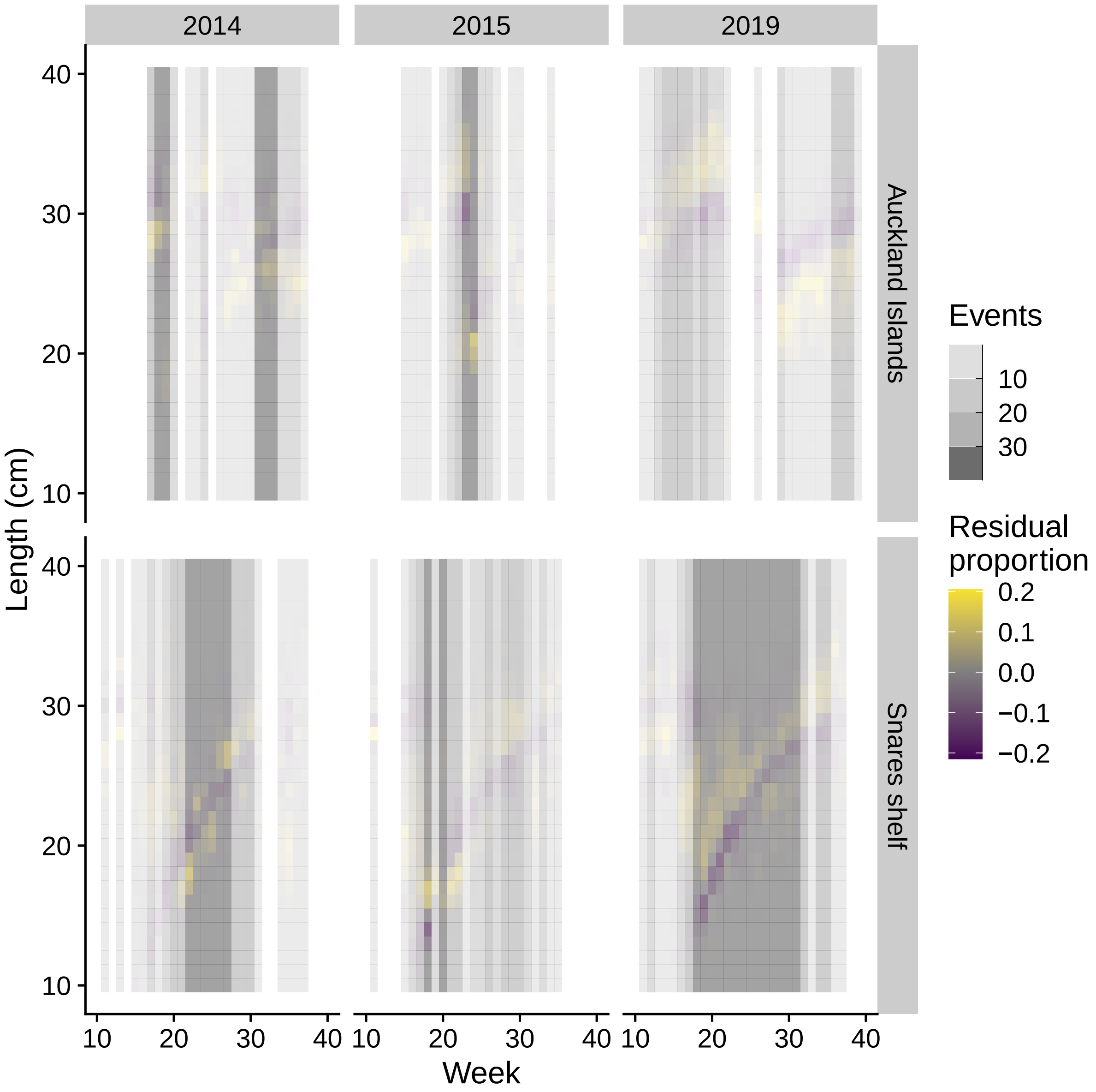

Figure 14: Residuals for length proportions by length and week for each of the two main areas and fishing years in the mixture model. Week 1 begins on 1 October each year; Snares shelf, Stewart-Snares shelf.

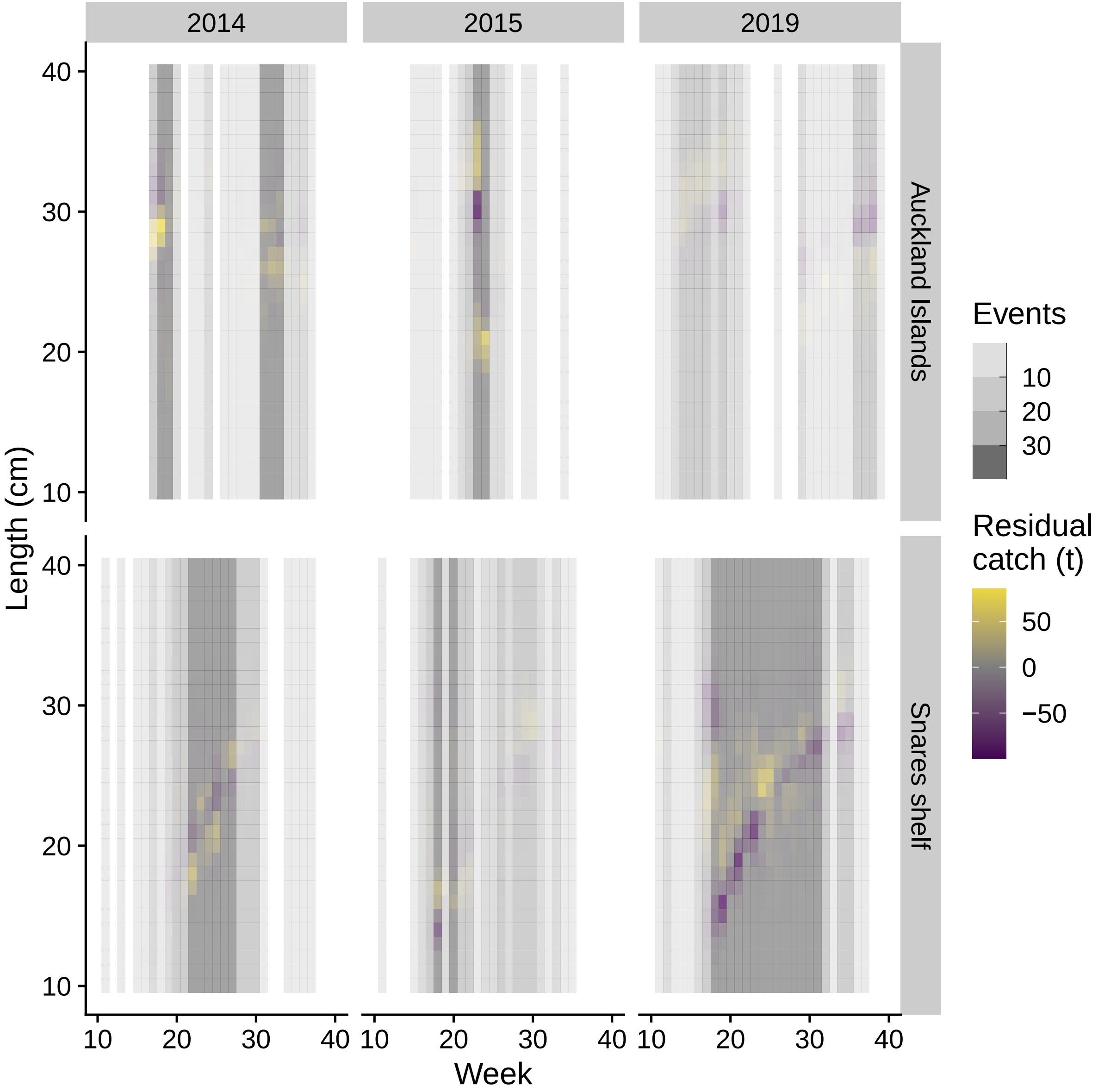

Figure 15: Residuals for catch-at-length by length and week for each of the two main areas and fishing years in the mixture model. Week 1 begins on 1 October each year; Snares shelf, Stewart-Snares shelf

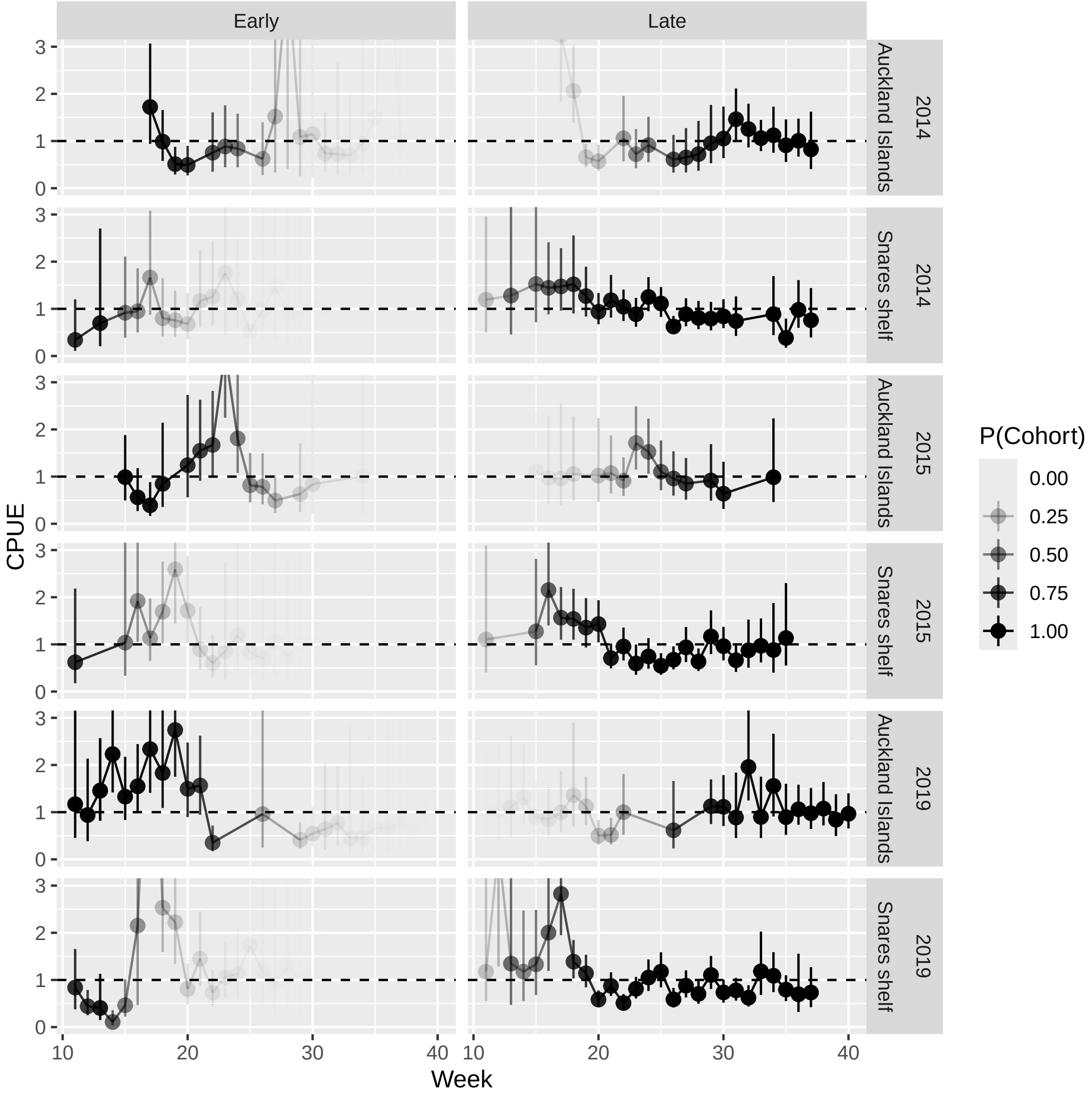

Figure 16: Within-season catch-per-unit-effort (CPUE) index for biomass by week for estimated early and late squid cohorts in the model, with shading indicating the proportional contribution of the cohort to catch in each week. Week 1 begins on 1 October each year; Snares shelf, Stewart-Snares shelf.

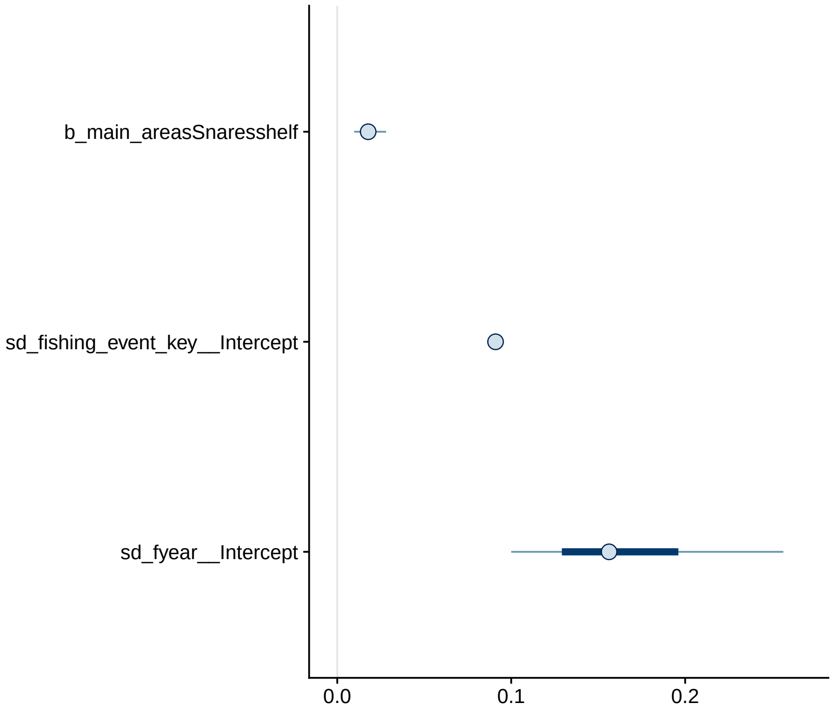

Figure 17: Effect size of area, fishing events, and fishing year in the length-weight model used to convert catch to numbers of squid in the catch-per-unit-effort-at-length mixture model.

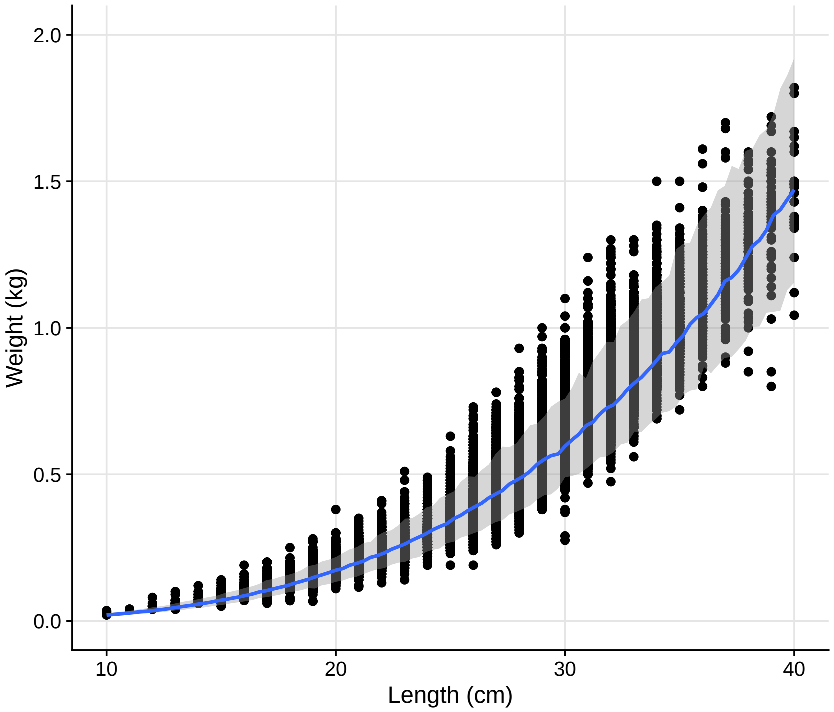

Figure 18: Estimated length-weight curve and 95% confidence used to convert catch to number of squid in the catch-per-unit-effort-at-length mixture model.

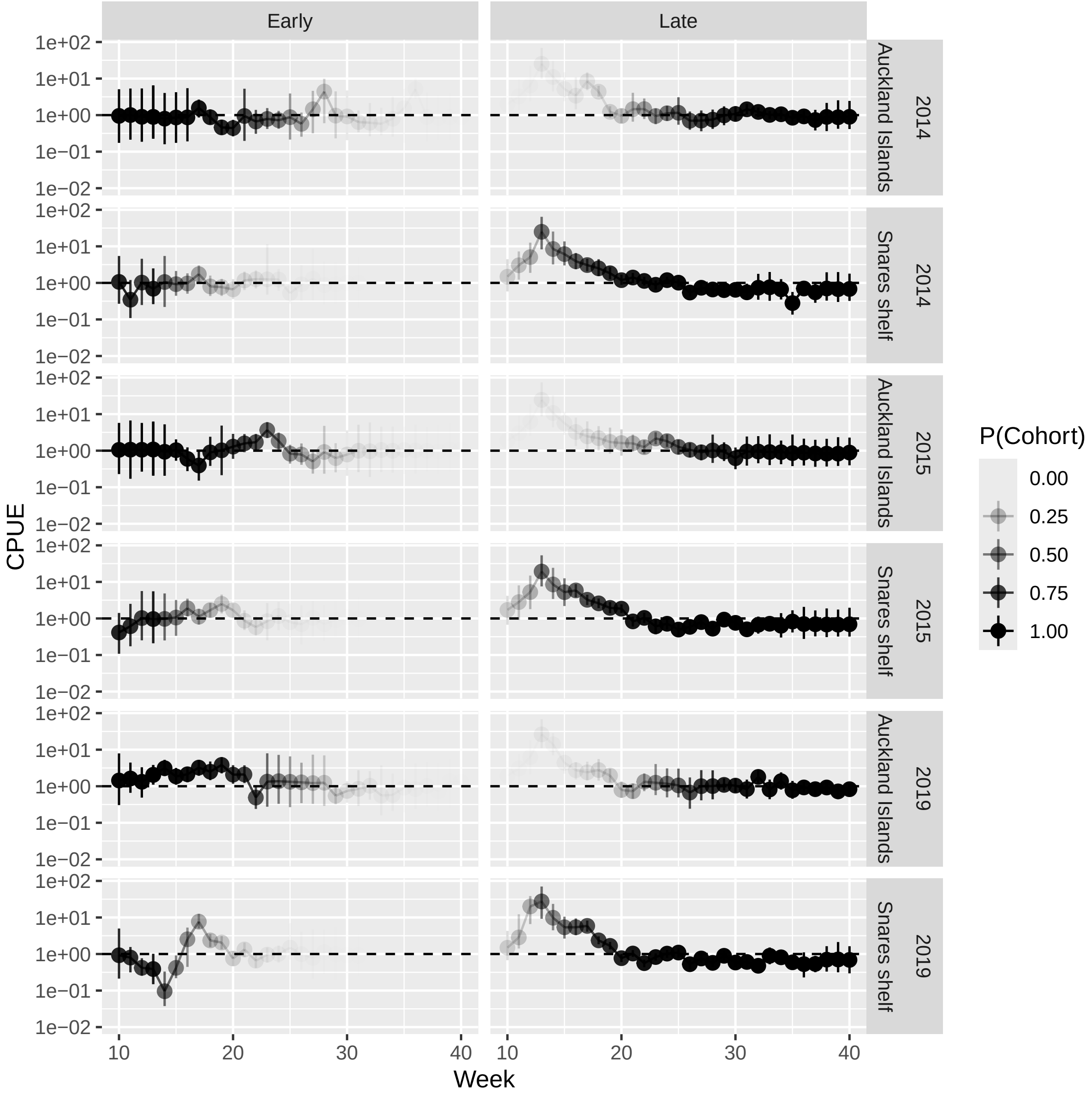

Figure 19: Within-season catch-per-unit-effort (CPUE) index for number of squid by week for estimated early and late cohorts in the model, with shading indicating the proportional contribution of the cohort to catch in each week. The index is plotted relative to a weighted geometric mean, where the weight is determined by the proportional contribution of each cohort in a given week. Week 1 begins on 1 October each year; Snares shelf, Stewart-Snares shelf.

4. DISCUSSION

The development of an integrated model to capture fishing on separate cohorts, and the temporal development of CPUE within seasons, provides the possibility to apply within-season DeLury depletion type models for squid in SQU 1T and SQU 6T. Although there are a number of improvements and extensions to the present model that would likely improve the accuracy of estimated trends by providing a closer match to data, the present model demonstrates the potential to develop these types of models by deriving indices of cohort abundance.

Patterns in CPUE suggested consistent depletion of numbers in the late cohort, while showing little consistency in trends for the early cohort in both areas. The early cohort was only consistently fished at Auckland Islands for the years considered in the present model. There was some evidence for declining numbers between weeks 15 and 20 in 2014 and 2015 in this first cohort. No such trend was evident for 2019. The second cohort was fished earlier on the Stewart-Snares shelf, and later at Auckland Islands. Nevertheless, CPUE trends were largely similar between both areas, and showed a marked reduction in numbers as squid grew from about 15 cm length in week 15 to about 20 cm length by week 20. These trends, along with broad changes in length that were similar across the southern Stewart-Snares shelf and at Auckland Islands over the following weeks, indirectly support the hypothesis of movement of small squid from the Stewart-Snares Shelf to Auckland Islands area.

There are a number of relatively straightforward extensions and technical improvements that we recommend be explored before the present model could be made operational for use in DeLury depletion estimates. Model extensions include:

- The model should be run over additional years. Due to the high computational cost of the model, this extension requires sufficient computing resources. It may be useful to split models into individual years; however, this split would mean losing the ability to learn about relevant effects across years, such as effects due to vessel differences or other standardising variables.
- CPUE derived from the present model (i.e., for a limited number of years) could be tested in a DeLury depletion model, particularly for the more steadily and heavily fished second cohort.
- Models with additional cohorts could be tested in years when residual patterns suggest the presence of an additional strong cohort.

Key technical improvements to further improve the model fit to data could also be explored:

- Uncertainty about catch-at-length should be better reflected in the model. Currently, length proportions are scaled to catch, which does not consider sampling error.
- The response variable should be modelled as a continuous variable (here, a negative binomial distribution was used to accommodate zero catch-at-length events, but this distribution was largely a choice of convenience). The Tweedie distribution, a combination of Poisson and Gamma distributions, would, at once, cover sampling error (a Poisson random variable over sampled length classes) as well as catch variability (Gamma distribution), and allow for zero catch in length classes; however, this distribution also adds an additional parameter and is considerably more difficult to implement.
- Random effects for annually-varying growth and recruitment timing of the second cohort appear warranted, but were not implemented here due the small number of years considered. Nevertheless, including these effects would allow the model to fit the residual patterns evident in the current analysis.
- The model could be directly fitted in numbers of squid rather than catch, by applying the length-to-numbers conversion prior to fitting the model. However, *a priori* it would appear that

this model would be equivalent to the present model in that the length-weight model is used to the same effect.

Operational use in the context of a DeLury depletion model would require application of the cohort-CPUE model on a continuous basis. In addition, it would need to be applied across the overall catch effort, not just observed effort, since catch from any event needs to be proportionally allocated to each cohort so that it is removed from the appropriate cohort in the DeLury model. Both of these aspects are readily achievable given timely catch-effort reporting over electronic reporting systems, and a two-stage application of the model. Given that not all effort is observed, and observer data are currently not available in real time, the full posterior probability of any event fishing on one or the other cohort ($P(Cohort|data)$), is not available at the same time as fishing information from catch and effort data. Nevertheless, in the present form, the model can still provide a “prior” probability for a fishing event targeting a given cohort in a given area and week using Equation 1. To improve this “prior”, the model could be extended to involve more detail in the linear predictor for cohort proportion (see Equation 1) if residuals point to greater spatial complexity in the transition between cohorts than was currently assumed. For example, finer spatial detail, such as splitting the Auckland Islands area into northwestern and eastern components, may improve prediction of cohort membership for individual fishing events given fine-scale variation in cohort availability early in the season (see Figures 5, 6). Once observer data for a given event area is available (i.e., currently with some weeks’ lag), the full posterior distribution of the model can be updated. This two-stage strategy is common in the estimation of time-series models such as Hidden Markov Models (HMMs; Scott (2002)) and related models. It takes advantage of the natural two-stage hierarchy of these models—as long as all variables attributing cohort membership in Equation 1 are available from catch effort data (e.g., detailed position if a more detailed spatial model is used for Equation 1), then the two-stage strategy is applicable.

A key assumption in the present model was that identified length-progression in the catch-at-length is due to growth of individual strong cohorts. An alternative hypothesis that cannot currently be discarded, is that the process is significantly more complex, with length-based schooling and continuous immigration (recruitment) and emigration (e.g., for spawning) of squid from areas where individuals of similar size school. The present model, for example, estimated growth parameters that are somewhat slower, and led to smaller squid than the prior based on estimates from statolith ageing (Uozumi & Ohara 1993, Uozumi 1998). Nevertheless, squid growth can be highly variable (Uozumi 1998), and estimates of asymptotic length in the models here were likely biased, caused by emigration from spawning aggregations of squid at larger sizes. In addition, the prior was derived in an ad-hoc way from Uozumi (1998), where growth was presented as a suite of logistic growth curves by spawning month. Visual comparisons of derived growth (see Figure 12) suggest our estimates are broadly compatible with existing growth estimates; careful consideration of existing growth data and estimates may improve the prior we used for the present model. Nevertheless, the present estimates were broadly consistent with the hypothesis of strong cohorts growing, with females emigrating to spawning grounds once they reach approximately 30 cm in mantle length.

While the present modelling provides a proof-of-concept rather than an operational tool, the modelling showed that significant progress can be made on detecting cohort structure by integrating catch-and-effort and length-composition data. The model, therefore, addressed some key difficulties encountered in previous attempts to fit DeLury methods to data for arrow squid in the southern waters of New Zealand (McGregor & Large 2016). Within the present framework, alternative formulations could be tested to ensure that the currently employed two-cohort hypothesis provides the best fit to the data on an annual basis. Uozumi (1998) suggested year-around spawning and recruitment for southern arrow squid. In some years, there may be more “strong cohorts”, or, alternatively, no strong spring and autumn cohorts, so that smaller recruitment events appear significant in the length proportions. It may be possible to formulate the present model to accommodate this “background” recruitment into it, so that only events that have clear length modes associated with dominant cohorts are attributed to those cohorts; in contrast, events with inconsistent length composition are treated as coming from a background distribution of minor cohorts. The overall approach, therefore, remains a plausible path towards the application of in-season DeLury depletion models, even in the context of more complex recruitment patterns between years.

5. ACKNOWLEDGEMENTS

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APPENDIX A: BAYESIAN MIXTURE MODEL FOR COHORTS

```
vBFmix <- bf(cfish ~ 1,
             theta2 ~ SW+main_areas,# + (1+SW||main_areas:fyear),
             nl = TRUE) +
  nlf(mu1 ~
lgamma((v+1)/2)-lgamma(v/2)+log((1+(length-eta1)^2)/(v*sigma^2))*(-(v+1)/2)
+  ffx + rfx) + # f(l) + std effects
  nlf(mu2 ~ lgamma((v+1)/2)-lgamma(v/2)+log((1+(length-eta2)^2)/(v*sigma^2))*(-(v+1)/2) + ffx
  nlf(eta1 ~ Linf*(1.0-exp(-(k)*(SW-tnot1)))) + # singe k and tnot for
cohort 1 - limtied observations
  nlf(eta2 ~ Linf*(1.0-exp(-(k*exp(kfx2))*(SW-tnot2)))) + # variable
tnot and k for cohort 2
  nlf(tnot2 ~ tnot1+tnot2pre*exp(tfx2)) + # force cohort 2 to start
after cohort 1 to enforce ordering
  lf(rfx ~ main_areas + (1|fyear) + (1|main_areas:fyear) + (1|SeasonWeek:main_areas:fyear)) +
  lf(rfx2 ~ main_areas + (1|fyear) + (1|main_areas:fyear) + (1|SeasonWeek:main_areas:fyear))
  lf(ffx ~ (1|fishing_event_key)+s(start_seabed_depth)+s(log(fishing_duration))+(1|vessel_key
  lf(kfx2 ~ (1|fyear)) +
  lf(tfx2 ~ (1|fyear)) +
  lf(Linf ~ 1,
     k ~ 1,
     v ~ 1,
     tnot1 ~ 1,
     tnot2pre ~ 1,
     sigma ~ 1)


# priors for each mixture component
priors =
  set_prior('normal(-26,10)',  nlpar = 'tnot1',ub=-20) + # force first
cohort  start before current fishing year (SW-20) to enforce mixture order
  set_prior('lognormal(log(26)-0.5^2/2,0.5)',  nlpar = 'tnot2pre',lb = 0) +
  set_prior('lognormal(log(0.2)-0.3^2/2,0.3)', nlpar = 'k') + # prior
on k from lit
  set_prior('lognormal(log(1)-1^2/2,1)', nlpar = 'v') +
  set_prior('normal(40,5)', nlpar = 'Linf',ub=50) + prior
on Linf from lit
  set_prior('normal(2,2.5)', nlpar = 'sigma',lb=0) +
  set_prior('normal(-2,2)', dpar = 'theta2',class = 'Intercept',lb=-5)
+ # fishing starts on cohort 1 (theta2_Int <0)
  set_prior('lognormal(log(5)-1^2/2,1)', dpar = 'theta2',class = 'b') +
  set_prior('constant(1)', nlpar='ffx',class='b', coef = "Intercept") +
  set_prior('constant(1)', nlpar='tfx',class='b', coef = "Intercept") +
  set_prior('constant(1)', nlpar='tfx2',class='b', coef = "Intercept") +
  set_prior('normal(0,5)', nlpar='ffx',class='b', coef = "slogfishing_duration_1") +
  set_prior('normal(0,5)', nlpar='ffx',class='b', coef = "sstart_seabed_depth_1") +
  set_prior('normal(0,5)', nlpar='rfx',class='b') +
  set_prior('normal(5,5)', nlpar='rfx',class='b', coef = "Intercept") +
  set_prior('normal(5,5)', nlpar='rfx2',class='b', coef = "Intercept") +
  set_prior('normal(0,5)', nlpar='rfx2',class='b') +
  set_prior('normal(0,2)', class='sigma1') +
  set_prior('normal(0,2)', class='sigma2') +
```

```
  set_prior('normal(0,2)', class='sd', nlpar='rfx') +
  set_prior('normal(0,2)', class='sd', nlpar='rfx2') +
  set_prior('normal(0,2)', class='sd', nlpar='ffx') +
  set_prior('normal(-0.1^2/2,0.1)', class='sd', nlpar='kfx2') +
  set_prior('normal(-0.1^2/2,0.1)', class='sd', nlpar='tfx2')


inits <- function() list(b_k = 0.1,
                         b_tnot1 = -34,
                         b_tnot2pre = 26,
                         sigma1 = 0.7,
                         sigma2=0.7,
                         b_sigma=2.5,
                         b_Linf = 30,
                         Intercept_theta2 = -3,
                         b_theta2 = c(2,0.3),
                         b_v = 0.075,
                         sds_ffx_1_1=0.001,
                         sds_ffx_2_1=0.001,
                         b_rfx=c(10,0),
                         b_rfx2=c(10,0),
                         bs_ffx=0,
                         sd_1 = 0.01,
                         sd_2 = 0.01,
                         sd_3 = 0.01,
                         sd_4 = 0.01,
                         sd_5 = 0.01,
                         sd_6 = 0.01,
                         sd_7 = 0.01,
                         sd_8 = 0.01,
                         sd_9 = 0.01,
                         sd_10 = 0.01,
                         sd_11 = 0.01,
                         sd_12 = 0.01
                         )



vBMix <- brm(vBFmix,
             data = catch_at_length,
             family = mixture(brmsfamily("negbinomial2"),
                              order = F,
                              nmix = 2),
             prior = priors,
             init = inits,
             chains = 4,
             cores = 64,
             iter=1000,
             threads = 64,
             backend='cmdstanr',
             refresh=10,
             adapt_delta = 0.95,
             max_treedepth = 12)
```